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Load-Deformation Behavior of Elastic Beam-Columns on Yielding Foundations[1]

The effect of an elastic-perfectly plastic foundation on compressive load carrying capacity of beam columns is considered. The configuration studied simulates a power plant condenser support plate acting as a column member to support the condenser walls against excessive deformation due to vacuum pressure. The yielding foundation represents the effect of the condenser tube bundle which can resist support plate bending through frictional action between the plate and the tubes. The equation governing lateral motion of the beam column is developed in rate form using the Green's function for an elastic beam column. An approximate incremental solution is obtained enabling the load-deformation history of the beam column to be studied incorporating elastic-plastic loading and unloading of the foundation. "Failure" is assumed to occur when the maximum stress in the beam column reaches a preset allowable design value.

Introduction

The increased need for power plant safety and reliability has recently led to additional emphasis on structural design of power plant condenser components [1].[2] Fig. 1 shows a section through a typical condenser with the support plate acting as a compressive load carrying member to inhibit side wall movement under vacuum loading. The support plate spacing is set to provide the required unsupported tube length necessary to minimize tube vibrations. The thickness of the support plate is then set by the compressive load magnitude necessary to support the walls under vacuum loading. Because of increased condenser size, unsupported lengths of the support plate are increasing; thus, not only must the direct compressive stress on the support plate be considered, but support plate instability under the compressive load must also be investigated. To inhibit lateral movement of the support plate due to buckling tendency under compressive loading, additional transverse bracing, shadowing the tube bundle, may be needed. It is desirable, however, to minimize additional structural bracing of the support plate because of flow interference; hence, in many configurations, it has been successfully argued that baffles and drain trays running the full length of the condenser provide the necessary lateral support. There are many units, operating successfully, where existence of such support from drain trays, etc., is questionable; hence, this may indicate that successful operation of such units is possible because the tube bundle itself provides lateral support to the tube support plates to inhibit an instability. Since it is desirable to establish rigorous criteria to predict the minimum amount of additional bracing needed in any specific case, the effect of the tube bundle in providing a portion of such bracing must be quantified.

Modeling of the configuration requires consideration that the tubes are permitted to slide relative to the tube support plates to accommodate thermal expansion. Due to manufacturing and erection tolerances, there is always some binding action occurring between tubes and support plates; hence, we consider that the tubes resist a given level of transverse movement as elastic springs and thereafter provide constant resisting force through frictional action. Such behavior is simulated by considering the tube bundle as an elastic-perfectly plastic foundation with the yield strength of the foundation related to the friction coefficient between tubes and tube support plate.

The literature relating to elastic plastic foundations reveals that prior work involving yielding supports has been restricted to vibratory

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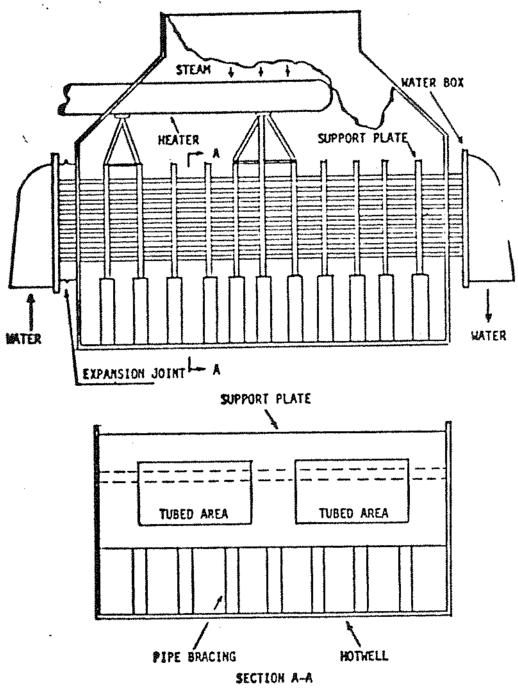


Fig. 1 Condenser arrangement

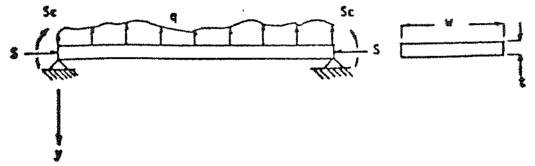


Fig. 2 Beam configuration

behavior of discrete systems; this class of systems is particularly amenable to numerical solution [2, 3]. Relatively few works have considered the behavior of continuous yielding systems. Reference [4] contains approximate solutions for the fundamental steady state response of a number of specific yielding systems. The problem to be studied herein involves a moving elastic-plastic interface which has been previously considered only in [5]. The problem considered in [5] deals with the steady-state response of a beam on a strain hardening foundation under a moving load. The formulation in [5] incorporates the effect of the moving elastic-plastic interface in the foundation, a feature that is also found in the work presented herein.

Theory

We consider the problem of predicting load deformation behavior of a simply supported beam, on an elastic-perfectly plastic foundation, subjected to compressive end loading. Because of the yielding foundation, the problem is not an eigenvalue problem; rather, a mechanism must exist for lateral movement to commence with the initiation of any end load. In practice, a mechanism is present either in the form of eccentrically applied end loading, or in the form of an initial beam deflection shape. The intent of the work herein is to focus attention on the subsequent behavior of the beam as the end load is increased in the presence of the yielding foundation. It should be noted that the simple beam model we consider herein with simply supported end conditions is a considerably idealized version of the actual condenser support plate. Observations of support plate deformations under compressive loading indicate that severe deformations, when they occur, are localized near the top of the support plates and are a combination of bending and twisting deformations. Our model herein assumes only bending deformations of a strip of plate; future efforts will consider the more realistic combined deformation mode.

We let w_T be the total lateral deformation of a rectangular beam strip (representing a section of support plate) such that

$$w_T(x) = w_0(x) + w(x) \tag{1}$$

where

$w_0(x)$ = initial deformation of beam caused by initial misalignment, etc.

$w(x)$ = deformation due to end loadings, etc.

We employ classical assumptions on smallness of center-line rotation of the beam, and obtain the following differential equation governing $w(x)$.

$$\frac{d^4w}{dx^4} + \frac{S}{EI}\frac{d^2w}{dx^2} = -\frac{q(w)}{EI} - \frac{S}{EI}\frac{d^2w_0}{dx^2} \tag{2}$$

In the foregoing equation S represents the compressive end load, and $q(w)$ represents the lateral load due to the foundation. Fig. 2 shows the beam model which yields the governing equation (2). The elastic-plastic character of the foundation is represented in incremental (rate) form as follows:

$$\begin{aligned} \dot{q} &= \bar{k}\dot{w} \begin{cases} |q| < q_{\mathrm{MAX}} & \\ \text{or } |q| = q_{\mathrm{MAX}} & \text{and } q\dot{w} < 0 \end{cases} \\ \dot{q} &= 0 \qquad |q| = q_{\mathrm{MAX}} \qquad \text{and } q\dot{w} > 0 \end{aligned} \tag{3}$$

The behavior given by equation (3) is shown in Fig. 3 for a typical location on the beam. The value of q_{MAX} depends on local geometry of the tube and tube support plate, and exhibits a strong dependence on the coefficient of friction. The "spring rate" $\bar{k}$ is a measure of the elastic resistance of the tubes prior to relative sliding. Associated with equation (2) are appropriate boundary conditions which include the effect of compressive load eccentricity.

Initially, for low load levels, the foundation behaves elastically over the entire beam length. At some value of end load, however, local foundation yielding occurs because of beam strip lateral deformation. Once slippage occurs, an interface between the elastic foundation region and the yielded region is established. The location and extent of the yielded region is a function of the particular configuration

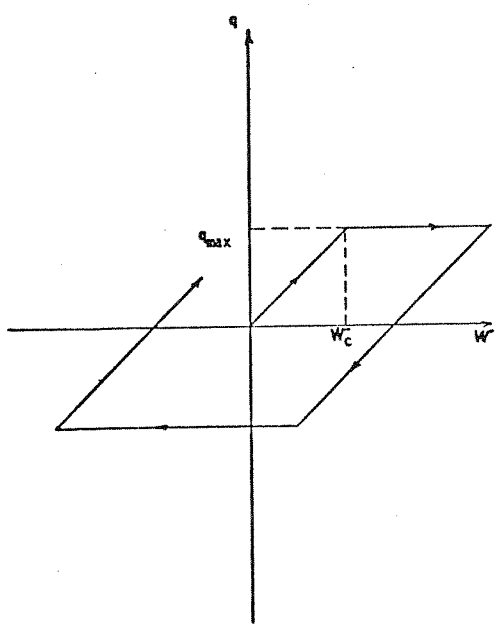


Fig. 3 Foundation characteristic

considered and of the current load level. To establish the proper load-deformation history of the beam, it is necessary to locate the elastic plastic interface at each stage of loading.

We consider in detail the case $w_0(x) = 0$ so that the mechanism for continuous lateral deformation of the beam, as end loading is applied, is provided by a small eccentricity ϵ of the applied load. As boundary conditions for equation (2), we specify

$$w(0) = w(L) = 0; \quad \left.\frac{d^2w}{dx^2}\right)_0 = \left.\frac{d^2w}{dx^2}\right)_L = -\frac{S\epsilon}{EI} \tag{4}$$

Because of the yielding foundation, the entire problem is best considered in rate form. Hence, the rate formulation of equation (2), (4) is ($w_0(x) = 0$)

$$\frac{d^4\dot{w}}{dx^4} + \frac{S}{EI}\frac{d^2\dot{w}}{dx^2} = -\frac{\dot{q}}{EI} - \frac{\dot{S}}{EI}\frac{d^2w}{dx^2}$$

$$\dot{w}(0) = \dot{w}(L) = 0; \quad \left.\frac{d^2\dot{w}}{dx^2}\right)_0 = \left.\frac{d^2\dot{w}}{dx^2}\right)_L = -\frac{\dot{S}\epsilon}{EI} \tag{5}$$

In equation (5), $\dot{w}$ represents the rate of change of displacement with respect to an arbitrary parameter, which may be end load or time. Given the displacement and stress distribution at any value of end load (or time), and the current state of the foundation, we solve equation (5) for the displacement rates, obtain from these the stress rates, and thus step forward in "time" to establish a new displacement and stress state. In the entire procedure, we assume that the beam itself remains elastic; only the foundation is subject to yielding and possibly elastic unloading, during the history of the deformation process.

Equation (5) is rewritten in integral form as

$$\dot{w} = \dot{p}_2(x) + \int_0^L \frac{G(x,\xi)}{EI}\left[-\dot{q}(\xi) - \dot{S}\frac{d^2w}{d\xi^2}\right]d\xi \tag{6}$$

where

$$\frac{d^2\dot{p}_2}{dx^4} + \frac{S}{EI}\frac{d^2\dot{p}_2}{dx^2} = 0$$

$$\dot{p}_2(0) = \dot{p}_2(L) = 0; \quad \left.\frac{d^2\dot{p}_2}{dx^2}\right)_0 = \left.\frac{d^2\dot{p}_2}{dx^2}\right)_L = -\frac{\dot{S}\epsilon}{EI} \tag{7}$$

and $G(x,\xi)$, a Green's function, satisfies

$$\frac{d^4G}{dx^4} + \frac{S}{EI}\frac{d^2G}{dx^2} = \delta(x-\xi) \tag{8}$$

$$G(0,\xi) = G(L,\xi) = \left.\frac{d^2G}{dx^2}\right)_{0,\xi} = \left.\frac{d^2G}{dx^2}\right)_{L,\xi} = 0$$

with $\delta(x-\xi)$ representing the Dirac Delta function.

The second term under the integral in equation (6) is integrated by parts twice, and after applying the boundary conditions on G and w, we obtain

$$\dot{w}(x) + \int_0^L G(x,\xi)\frac{k^*\dot{w}}{EI}(\xi)d_\xi = \dot{p}_2(x) - \frac{\dot{S}}{EI}\int_0^L \frac{\partial^2 G}{\partial\xi^2}w(\xi)d\xi \tag{9}$$

where we have employed equation (3) and assumed that $k^* = \bar{k}$ or 0 depending on the current local state of the foundation.

It is easily shown that equations (7) and (8) yield

$$\dot{p}_2(x) = \frac{\dot{S}\epsilon}{EIk^2}\,\frac{\sin kx + \sin k(L-x) - \sin kL}{\sin kL}; \qquad k^2 = \frac{S}{EI} \tag{10}$$

$$G(x,\xi) = \begin{cases} -\dfrac{1}{k^2}H(x,\xi) + \dfrac{\sin k(L-\xi)\sin kx}{k^3\sin kL} & x \le \xi \\[2ex] -\dfrac{1}{k^2}H(x,\xi) + \dfrac{\sin k(L-x)\sin k\xi}{k^3\sin kL} & x \ge \xi \end{cases} \tag{11}$$

where

$$H(x,\xi) = \begin{cases} \dfrac{L-\xi}{L}x & x \le \xi \\[2ex] \dfrac{L-x}{L}\xi & x \ge \xi \end{cases} \tag{12}$$

Also, it can be shown from equation (11) that

$$\frac{\partial^2 G}{\partial \xi^2} = -H(x,\xi) - k^2G(x,\xi) = \frac{\partial^2 G}{\partial x^2} \tag{13}$$

Hence, substituting equation (13) into (9) we write

$$\dot{w}(x) + \int_0^L \frac{G(x,\xi)}{EI}k^*\dot{w}d\xi = \dot{p}_2(x) + \frac{\dot{S}}{EI}\int_0^L [H(x,\xi) + k^2G(x,\xi)]w(\xi)d\xi \tag{14}$$

The maximum stress rate at any cross section is

$$\dot{\sigma} = \frac{\dot{S}}{A} + \left|\frac{\dot{M}C}{I}\right|; \qquad \frac{\dot{M}}{EI} = -\frac{d^2\dot{w}}{dx^2} \tag{15}$$

where A is the cross-sectional area, I = second moment of inertia and C herein is equal to one half the support plate thickness. Using the previous results, we obtain an expression for the stress rate $\dot{\sigma}$ as

$$\dot{\sigma} = \frac{\dot{S}}{A} + \left|\frac{\dot{S}}{A}\frac{\epsilon C}{r^2}\phi(x) - \int_0^L \left\{\frac{C}{r^2}[H(x,\xi) + k^2G(x,\xi)] \times \left[\frac{k^*}{A}\dot{w}(\xi) - \frac{k^2\dot{S}}{A}w(\xi)\right]\right\}d\xi\right| \tag{16}$$

where we define $r = (I/A)^{1/2}$ = the radius of gyration of the beam section, and

$$\phi(x) = \frac{\sin kLy + \sin kL(1-y)}{\sin kL}; \qquad y = x/L \tag{17}$$

We define the dimensionless quantities

$$\eta = \xi/L; \qquad u = w/\epsilon; \qquad p = kL \tag{18}$$

$$\frac{G(x,\xi)}{L^3} = \begin{cases} g^1(y,\eta) = -\dfrac{1}{p^2}h(y,\eta) + \dfrac{\sin p(1-\eta)\sin py}{p^3\sin p} & y \le \eta \\[2ex] g^2(y,\eta) = -\dfrac{1}{p^2}h(y,\eta) + \dfrac{\sin p(1-y)\sin p\eta}{p^3\sin p} & y \ge \eta \end{cases} \tag{19}$$

$$H(x,\xi)/L = h(y,\eta) = \begin{cases} (1-\eta)y & y \le \eta \\ (1-y)\eta & y \ge \eta \end{cases} \tag{20}$$

We introduce the rates of change of displacement and stress with respect to load S, $\partial u/\partial S$, $\partial\sigma/\partial S$, and obtain the final set of rate equations as

$$\frac{\partial u}{\partial S} + \int_0^1 \beta^2 g(y,\eta)\frac{\partial u}{\partial S}d\eta = \frac{L^2}{EI}\left\{\frac{(\phi(y)-1)}{p^2} + \int_0^1 [p^2g(y,\eta) + h(y,\eta)]u(\eta)d\eta\right\} \tag{21}$$

$$\frac{\partial\sigma}{\partial S} = \frac{1}{A} + \frac{\epsilon C}{r^2}\left|\frac{\phi(y)}{A} - \int_0^1 \{h(y,\eta) + p^2g(y,\eta)\}\left\{\frac{\beta^2Er^2}{L^2}\frac{\partial u}{\partial S} - \frac{p^2}{A}u(\eta)\right\}d\eta\right| \tag{22}$$

where

$$\beta^2 \equiv k^*L^4/EI$$

A similar set of equations (Appendix A) can be developed for the case where the driving mechanism for lateral plate motion is an initial plate displacement shape rather than load eccentricity.

Incremental Solution Procedure

With stress and displacement rates known at every location for the current value of load S, then displacement and stress at $S + \Delta S$ are determined by the incremental scheme

$$\begin{aligned} U(S+\Delta S) &= U(S) + (\partial u/\partial S)\Delta S \\ \sigma(S+\Delta S) &= \sigma(S) + (\partial\sigma/\partial S)\Delta S \end{aligned} \tag{23}$$

The load increment on the plate due to the elastic plastic foundation is given from equation (3) as

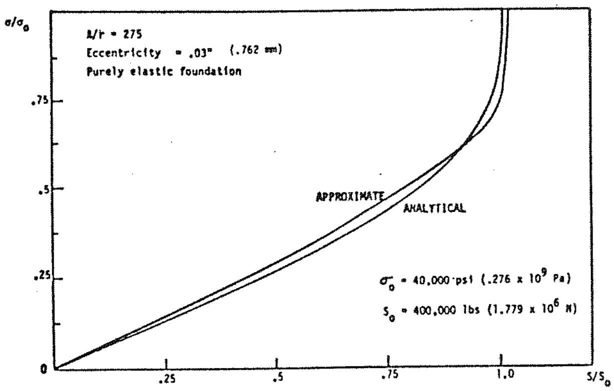


Fig. 4 Comparison between approximate method and analytical method

$$q(S + \Delta S) = q(S) + (\partial q/\partial S)\Delta S$$

where

$$\partial q/\partial S = k^*\epsilon \frac{\partial u}{\partial S} = \beta^2 \frac{EI\epsilon}{L^4}\frac{\partial u}{\partial S} \tag{24}$$

and

$$k^* = \bar{k} \qquad \text{for } |q| < q_{\max}$$
$$\text{or } |q| = q_{\max} \qquad \text{and} \qquad q(\partial u/\partial S)\Delta S < 0$$
$$k^* = 0 \ |q| = q_{\max} \qquad \text{and} \qquad q(\partial u/\partial S)\Delta S > 0 \tag{25}$$

The integral equations are solved approximately by discretizing the integrals in an appropriate manner; hence, we obtain rate solutions only at j discrete points along the length. The elastic or plastic action of the foundation is characterized by zero or nonzero values of $\beta_j{}^2$ at these specific locations. The critical deflection $\bar{V}_c$ at which the foundation first yields is given by

$$\bar{V}_c = \left|\frac{q_{\max}}{k^*\epsilon}\right| \tag{26}$$

Either equation (26) or current values of q are used to establish initial "failure" of the foundation along the beam length.

The discretized displacement rate equation yields the equation set

$$\left.\frac{\partial u}{\partial S}\right)_i + \sum_{j=1}^{n} D_j\beta_j{}^2 g(y_i, \eta_j)\left.\frac{\partial u}{\partial S}\right)_j = \frac{L^2}{EI}\left\{\frac{\phi - 1}{p^2} + \sum_{j=1}^{n}\left[p^2 g(y_i, \eta_j) + h(y_i, \eta_j)\right]D_j u(\eta_j)\right\} \tag{27}$$

$$i = 1, 2, \ldots n$$

where D_j are integration weighting coefficients. A similar set of equations is obtained for the variables $\partial\sigma/\partial S)_i$. For the initial load interval $S = 0$, we use appropriate limiting values of the Green's functions and other functions.

The incremental solution process is continued until a load level is obtained where the displacement $u_{\bar{k}}$ at one station $\bar{k}$ reaches the critical level $\bar{V}_c$ given by equation (26). In the subsequent interval, the parameter $\beta_k{}^2$ associated with this location of the $\bar{k}$th station is set to zero. The incremental solution continues until the load level is reached where adjacent stations indicate a yielding value for u, etc.

At each loading stage, we need to determine the appropriate value for $\beta_j{}^2$ at station j. Assume that we have reached a loading stage such that M foundation locations are yielding in their current state. To compute the current value of $\partial u/\partial S)_i$ for the next load increment, we set $\beta_m{}^2 = 0$, $m = 1, 2, \ldots M$ in equation (27) and solve the resulting algebraic equations. At each of the m yielded stations, we examine the sign of the product.

$$P_m = q_m \left.\frac{\partial u}{\partial S}\right)_m \Delta S \qquad m = 1, 2, \ldots M \tag{28}$$

Initially, we assume $\Delta S > 0$ for the next increment. If all $P_m > 0$ $m = 1, 2, \ldots M$, the values of $\beta_m{}^2$ are consistent and the values $\partial u/\partial S)_i$ establish deflection state at the end of the interval. However, if one or more $P_m < 0$, then the assumed values of $\beta_m{}^2$ are inconsistent and the calculation for $\partial u/\partial S)_i$ is repeated with new values for $\beta_m{}^2$ indicated by the current sign of P_m. The iteration on $\beta_m{}^2$ and P_m continues until consistency is achieved at all stations. In the symmetrically loaded problems considered herein, it is found that for certain configurations a state of loading is reached where consistency between the choice of $\beta_m{}^2$ and the resulting value of the product P_m is only achieved if the load increment ΔS is negative for the next increment. The tendency for the load to decrease while the deflection continues to increase is indicative of "snap-thru" behavior. The presence of this "snap thru" effect is associated with the beam strip deflection shape undergoing a gross change in form.

Numerical Results

We initially compare the approximate incremental solution, based on the discretized equation (27), with an exact solution. The results of an exact solution and our approximate solution are compared in Fig. 4 where the extreme fiber stress at $x = L/2$ is plotted against direct load S. Five points are considered in the discretization of equation (31). The approximate solution predicts a "failure" load within 4 percent of the exact analytical solution.

Figs. 5 and 6 show, for a typical set of parameters, the direct stress $\sigma_c = S/A$ versus $\bar{V}$, a dimensionless measure of the total lateral movement of the center point of the beam. The deformation measure $\bar{V}$ is taken as

$$\bar{V} = \sum_{r=1}^{N}\left|\Delta u^{(r)}\right| = \sum_{r=1}^{N}\left|\left.\frac{\partial u}{\partial S}\right)^{(r)}\Delta S^{(r)}\right| \tag{29}$$

where the sum is taken over N load increments. Fig. 5 shows typical results for the beam loaded eccentrically, while Fig. 6 shows typical results for the beam with an initial deflection. The parameter $\epsilon = 0.03$ in. (0.762 mm) for both cases. For values of the critical displacement $\bar{V}_c$ considered, the load first increases with increasing deflection, and then decreases. Eventually, the load again begins to increase; however, in the cases considered, it is found that the beam fiber stress, in the interior, reaches unacceptable levels at some point while the load is decreasing from its maximum value. Hence, from a design point of view, the initial maximum load point, prior to load decrease, is considered as the maximum acceptable load for the given beam. Note that in Figs. 5 and 6, the beam slenderness ratio is representative of a relatively weak member from the point of view of column stability in the absence of any foundation. A quite different behavior is shown for a beam with a lower slenderness ratio (Fig. 7). In this case, no "snap-thru" behavior appears, and the direct loading continues to rise to its limiting value which may be set either to limit central deflection, or to limit maximum extreme fiber stress.

The differences in behavior of load deflection curves for large and small slenderness ratio are explained in qualitative terms by Figs. 8 and 9 where the dimensionless beam mode shape is plotted at different load levels. It is seen that the rising and then falling load curve, associated with the larger slenderness ratio, is related to deformation shape changes. For the smaller slenderness ratio, however, no such change of shape occurs as the load increases.

Fig. 10 shows the effect of L/r ratio on maximum allowable load for values of $\bar{V}_c$ varying from zero to infinity. The results for $\bar{V}_c = 0$ correspond to the well known results for an initially deformed beam without any foundation. The curves corresponding to $\bar{V}_c = \infty$ correspond to the beam strip under a purely elastic foundation. The dip in the allowable load curve in this case is most likely due to the interaction between the driving mechanism of load eccentricity or initial deformation and the resisting effect of the purely elastic foundation. The curves for elastic plastic foundation fall between these limiting curves. The dotted curve shown in the figures essentially marks the demarcation between single lobe final deflection shape (Fig. 9(*a*), 9(*b*),

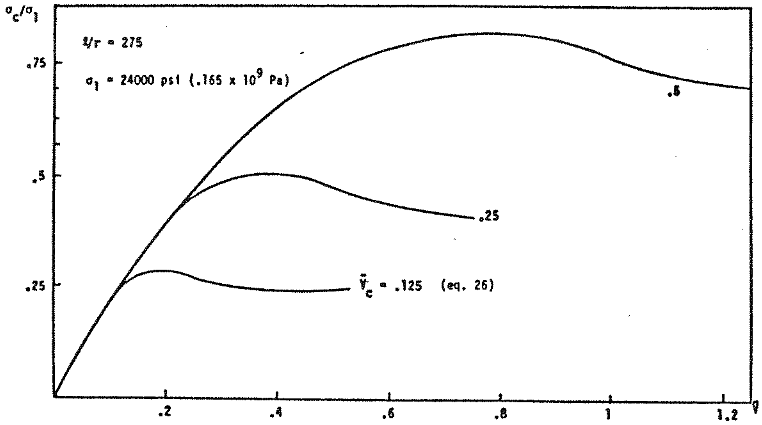


Fig. 5 Load-deformation curve for beam under thrust and end moments

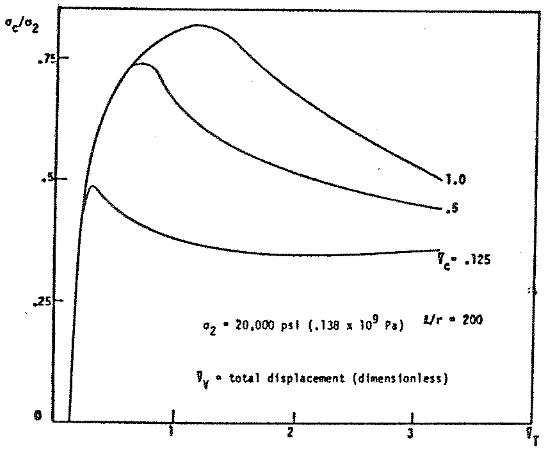


Fig. 6 Load deformation for a beam with initial curvature

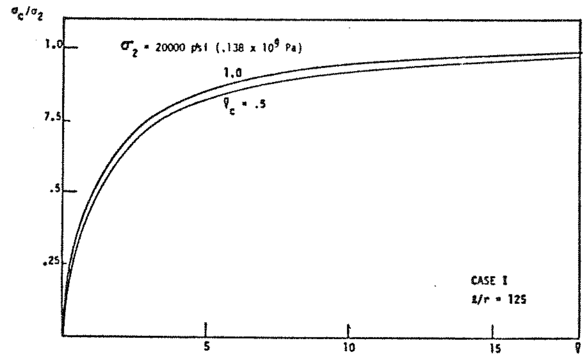


Fig. 7 Load-deformation curve for beam with small slenderness ratio

and multiple lobe deformation shape (Fig. 8(a), 8(b). Generally, the value σ_c for points to the left of the dotted line corresponds to load S at which the interior beam stress reaches a maximum value (36000 psi (248 MPa)) without any "snap-thru" behavior; the value σ_c for points to the right of the dotted line correspond to the maximum load S achievable prior to "snap-thru." The characteristic snap-thru behavior of the load may not appear for the case $\overline{V}_c = \infty$; the presence of the dotted line through this curve merely indicates the character of the final deformation shape. It should also be emphasized that the plotting of the dotted curve in Fig. 10 is based solely on a series of computer runs and is represented on the curves solely for informational purposes. All of the figures show results for a *particular* strip configuration and do not necessarily reflect any generalized curve.

Typical Design Computation

In the preceding section, some typical results are presented without regard to the relation of the configuration studied to any tube support plate-tube configuration. Herein, we consider a set of representative data indicative of a large condenser unit and establish the maximum compressive load capacity of a representative section of a tube support plate.

Tube Geometry and Properties. O.D. = 1.25 in. (31.75 mm); thickness = 0.035 in. (0.889 mm) Young's modulus = 1.7×10^7 psi (11.72×10^4 mPa); half length of condenser = 276 in. (7010 mm).

Beam Strip Geometry and Properties. Thickness = 0.5625 in. (14.29 mm); width = 30 in. (762 mm); Length = 120 in. (3048 mm); effective Young's modulus (deflection efficiency = 0.25) = 7.5×10^6 psi (51.7×10^4 MPa); distance between support plates = 32 in. (813 mm); number of tubes/inch (in a 30-in. width) = 19; coefficient of friction = 0.2.

We note that each tube (between adjacent support plates) has a total weight when filled with water equal to W_t. For the configuration at hand

$$W_t = 2.4608 \text{ lb/tube } (10.95 \text{ N/tube})$$

The elastic spring constant for a single tube is conservatively estimated by considering a tube of length equal to the total condenser length, loaded by a unit longitudinal load at the center. This yields the following result:

$$K_T = \frac{2A_TE_T}{L_T} \quad (30)$$

where A_T, E_T, L_T are the tube metal area, Young's modulus, and half length, respectively. For the configuration at hand, we obtain

$$K_T = 1.6458 \times 10^4 \text{ lb/in. } (290 \text{ N/m}) \quad (31)$$

With nineteen tubes per inch specified, we obtain the elastic foundation modulus needed for equation (3) as

$$\bar{k} = 19\, K_T = 31.27 \times 10^4 \text{ psi } (2156 \text{ MPa}) \quad (32)$$

The critical deflection for slippage of a tube is computed assuming that μW_T is the maximum load that can be supported, with μ being the coefficient of friction. Thus, we have, for the dimensionless critical displacement $\overline{V}_c$

$$\overline{V}_c = \frac{w_c}{\epsilon} = \frac{\mu W_T}{\epsilon K_T} = \frac{0.299 \times 10^{-4}}{\epsilon} \tag{33}$$

For a driving mechanism to force lateral displacement of the beam upon initiation of the end compressive load, we consider the case of an initially deformed beam in accordance with equation (A-3) of Appendix A. For a maximum initial deflection of 0.125 in. (3.2 mm), we have ϵ = 1 in. (25.4 mm).

The above computations provide the input for the computer simulation discussed herein The final results indicate that the maximum compressive load that is sustained by the beam strip is

$$S_{MAX} = 110{,}500 \text{ lbs } (4.95 \times 10^5\ N) \tag{34}$$

The "failure" condition for this geometry is a "snap-thru" condition similar to Fig. 6 with a multilobed final deflected state. Five stations are used in the computations. The beam fiber stress at maximum load is

$$\sigma = 8226 \text{ psi } (56.7 \text{ MPa}) \tag{35}$$

Since this stress is for an equivalent solid beam, the actual ligament stress (assuming a stress increase of 4) is

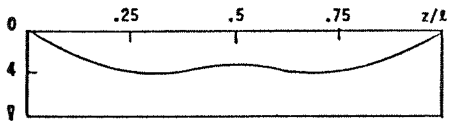
$$\sigma \text{ (ligament)} = 32904 \text{ psi } (226.8 \text{ MPa}) \tag{36}$$

In order to assess the effect of the yielding foundation in this particular design case, we consider two limiting cases. First we consider the strip with no initial deformation and no foundation effect. The Euler buckling load is

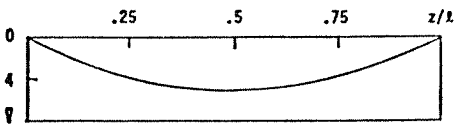
$$S_e = \pi^2 EI/L^2 = 2287 \text{ lbs } (10173 \text{ N}); \qquad \overline{E} = \begin{matrix}\text{effective}\\ \text{modulus}\end{matrix} \tag{37}$$

CASE I: ℓ/r=275, $\overline{V}_c$ = 4.0

Ecc. = .03" (.762 mm)



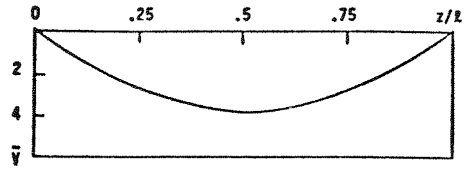
(A) BEAM MODE SHAPE AT "MAXIMUM LOAD"
$s = 3.68 \times 10^5$ lb $(16.37 \times 10^5$ N)



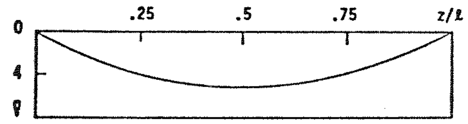
(B) BEAM MODE SHAPE AT A POINT ON UNSTABLE PART OF THE LOAD-DEFORMATION CURVE
$s = 3.56 \times 10^5$ lb $(15.84 \times 10^5$ N)

Fig. 8

CASE I: ℓ/r=125, $\overline{V}_c$ =4.0, Ecc. = .03" (.762 mm)



(A) BEAM MODE SHAPE AT YIELDING LOAD
$s = 3.38 \times 10^5$ lb. $(15.03 \times 10^5$ N)



(B) BEAM MODE AT A LOAD HIGHER THAN YIELDING LOAD
$s = 3.49 \times 10^5$ lb $(15.52 \times 10^5$ N)

Fig. 9

Note that the ratio of unsupported length to radius of gyration is L/r = 739 so that we ordinarily would not expect this strip to be an effective compression member in the absence of any foundation.

Now, consider the same strip assuming that the foundation provided by the tubes is maintained in the elastic regime. The critical buckling load for this case is expressed by

$$S = S_e\left[n^2 + (L/\pi)^4 \frac{k}{n^2\overline{E}I}\right]; \qquad n = \text{wave number} \tag{38}$$

For the configuration studied, the minimum value of the term in

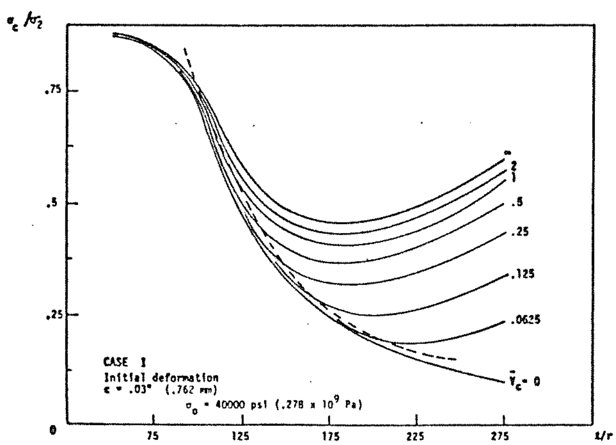


Fig. 10 Average compressive load versus slenderness ratio

brackets is 893 and occurs for $n = 21$. Thus, the buckling load in the presence of a completely elastic foundation is

$$S = 2{,}042{,}291 \text{ lbs } (9.1 \times 10^6 \text{ N}) \quad (39)$$

which is far in excess of that required to cause yielding in the strip. Comparing equations (34), (37), and (39), it is readily apparent that the effect of the tubes, even though deformation is resisted only by frictional action, is sufficient to increase the allowable load by a factor of approximately 50.

Conclusions

A design method is presented to account for the effect of the tubes, acting as a resisting foundation, on the load carrying capacity of power plant condenser tube support plates. The effect of the tubes is simulated by an elastic-plastic foundation with yield strength determined by the coefficient of friction between tubes and support plate. Only beam bending has been considered herein; the case of combined bending and twisting of the beam strip will be reported on in a subsequent work. Also left for a future study is the incorporation of the effect of tube-tube hole clearance on the magnitude of the critical deformation governing the onset of sliding. It is clear that once the tube support plate has rotated locally through a sufficient angle (depending on initial clearances), the bending strength of the tubes causes increased normal contact forces between tube and tube support plate. These increased normal forces imply that the critical deflection required for local slippage is also increased. Experimental verificaton of the design method has been obtained; the experimental procedure and the verification analysis will also be reported in a subsequent paper. It is noted that the results reported herein, and the additional efforts alluded to above, are presented in complete detail in the final EPRI report of contract RP-372-1 [6].

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APPENDIX

Beam Subject to Initial Deformation

We consider a simply supported strip with an initial deformation $w_0(x)$ characterizing a manufacturing or erection deviation from a perfect strip. The differential equation and boundary conditions for the subsequent deflection $w(x)$ due to axial compressive force S applied at the beam centerline is

$$\frac{d^4w}{dx^4} + \frac{S}{EI}\frac{d^2w}{dx^2} = -\frac{q(w)}{EI} - \frac{S}{EI}\frac{d^2w_0}{dx^2}$$

$$w(0) = w(L) = \left.\frac{d^2w}{dx^2}\right)_0 = \left.\frac{d^2w}{dx^2}\right)_L = 0 \quad (A\text{-}1)$$

In rate form the equation may be written

$$\frac{d^4\dot{w}}{dx^4} + \frac{S}{EI}\frac{d^2\dot{w}}{dx^2} = -\frac{\dot{q}}{EI} - \frac{\dot{S}}{EI}\frac{d^2w_0}{dx^2} - \frac{\dot{S}}{EI}\frac{d^2w}{dx^2}$$

$$\dot{w}(0) = \dot{w}(L) = \left.\frac{d^2\dot{w}}{dx^2}\right)_0 = \left.\frac{d^2\dot{w}}{dx^2}\right)_L = 0 \quad (A\text{-}2)$$

Suppose the initial deflection can be written

$$w_0(x) = \frac{\epsilon}{2L^2}x(L-x) \quad (A\text{-}3)$$

Then the integral equations for the rates corresponding to equations (25) and (26) are

$$\frac{\partial u}{\partial S} + \beta^2 \int_0^1 g(y,\eta)\frac{\partial u}{\partial s}(\eta)d\eta$$

$$= \frac{L^2}{EI}\left\{T_1(y) + \int_0^1 [h(y,\eta) + p^2g(y,\eta)]u(\eta)d\eta\right\} \quad (A\text{-}4)$$

$$\frac{\partial\sigma}{\partial S} = 1/A + \frac{\epsilon C}{r^2}\Bigg| - \frac{1}{A}T_2(y)$$

$$+ \int_0^1 [h(y,\eta) + p^2g(y,\eta)]\left[\frac{\beta^2r^2E}{L^2}\frac{\partial u}{\partial S} - \frac{p^2u(\eta)}{A}\right]d\eta\Bigg| \quad (A\text{-}5)$$

where

$$T_1(y) = \frac{\sin py + \sin p(1-y) - \sin p}{p^4 \sin p} - \frac{y(1-y)}{2p^2} \quad (A\text{-}6)$$

$$T_2(y) = \frac{1}{p^2}\left(\frac{\sin py + \sin p(1-y)}{\sin p} - 1\right) \quad (A\text{-}7)$$