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K.P. Singh
President & CEO
Holtec International
Marlton, NJ

Optimization of Two Stage Evaporators for Minimizing Rad-Waste Entrainment

Minimizing solute carry-over into the product is the central object in the design of Radioactive Waste Concentration Systems. Two Stage Evaporators are intrinsically superior to single stage designs in achieving this objective. An analysis of two stage system to optimize its performance is herein developed. A numerical example is employed to illustrate major conclusions.

1 Introduction

In the long history of industrial evaporation processes, continuous evaporations have often been performed in two stages. In most cases, these have been done on liquors that thicken at high concentrations. When the evaporator is of a circulating type, which is nearly always the case, the heat transfer medium is the concentrated solution itself. By evaporating in two stages, most of the water can be boiled out of thin liquor under good heat transfer conditions. The final boiling to the desired concentration is subsequently done in a small evaporator called a "finishing pan." This arrangement is not only economical of heat transfer equipment, but the loss of product by entrainment to the condenser is reduced, since most of the vapor evolves from liquor that is of much lower concentration than the final product.

When evaporating solutions of radioactive wastes as is done in nuclear power plants, the whole purpose of the evaporator is for de-contamination of the water portion and disposal of the concentrated solids. The entrainment of solids to the condenser is then the main design consideration; the objective is to produce a pure distillate. For this purpose two-stage evaporation is very effective. However, its effectiveness has not been well recognized in the nuclear industry. We shall show that the total entrainment of solids to the distillate can be greatly reduced by providing a second stage evaporator that is usually very small. We provide here also a method for determining the optimum division of evaporation duty between the two stages to effect the least carry-over. This method will also calculate the correct intermediate concentration based upon the relative deentrainment efficiencies in the vapor separation of the two evaporators.

2 Analysis

It is reasonable to suppose that, within certain limits, the concentration of entrained solids that are contained in the vapor is proportional to the concentration of solids in the liquid from which it boils, since these solids are in the droplets of liquid that is entrained. This may not be strictly true over a wide range of concentrations due to changes in properties that affect entrain-

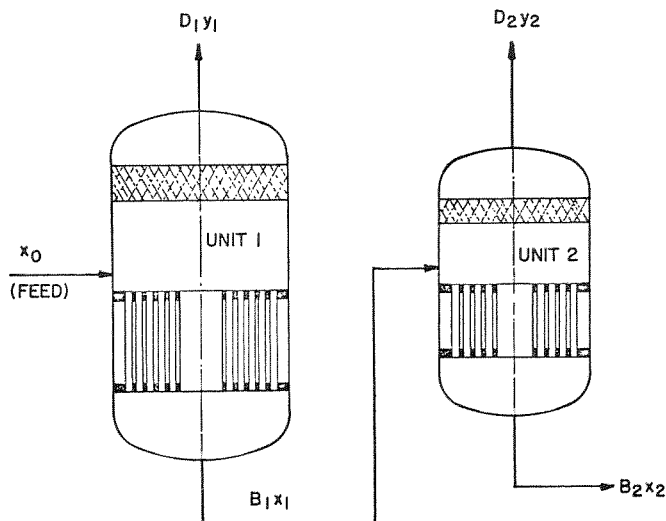


Fig. 1 Flow diagrams of solids in a two-stage evaporation

ment, namely viscosity, density, and surface tension. For the mathematical treatment that follows, we assume that this proportionality holds within the range of concentrations that applies to each of the two stages individually. The proportionality factors thus are different for the two stages, this difference reflecting also the disengagement efficiencies of the two evaporators.

Thus, if x_1 and x_2 denote the solute concentrations in the two evaporators, then the respective vapor concentrations y_1 and y_2 are given by

$$\begin{aligned} y_1 &= \alpha_1 x_1 \\ y_2 &= \alpha_2 x_2 \end{aligned} \quad (1)$$

where α_1 and α_2 are "de-entrainment factors" for the two evaporators. Furthermore, let x_0 denote the solute concentration in the feed. The initial concentration x_0 and final concentration of the concentrate, x_2 , are known quantities in the physical problem. The object now is to determine the intermediate concentration x_1 such that the total solute carry-over in the distillate, ϕ , is minimized. If D_1 and D_2 denote the amounts of distillate evaporated in Units 1 and 2 respectively, then

Contributed by the Design Automation Committee for presentation at the Design Engineering Technical Conference, St. Louis, Mo., Sept. 10-12, 1979 of the AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters Mar. 1, 1979. Paper No. 79-DET-26.

$$\phi = D_1 y_1 + D_2 y_2 \quad (2)$$

It should be noted that the solids concentration in the distillate, ψ , is given by

$$\psi = \frac{\phi}{D_1 + D_2} \quad (2a)$$

It is shown in the appendix that the minimum of ψ and ϕ functions occur at the same value of x_1 .

Referring to Fig. 1, solute balance in Unit #1 yields

$$x_0 = D_1 y_1 + B_1 x_1 \quad (3)$$

where the feed rate is assumed to be unity without any loss of generality. Equations (1) and (3) yield

$$D_1 = \frac{x_0 - x_1}{(\alpha_1 - 1)x_1} \quad (4)$$

Solute balance in Unit #2 yields

$$B_1 x_1 = D_2 y_2 + B_2 x_2 \quad (5)$$

We also note that

$$B_2 = 1 - D_1 - D_2 \quad (6)$$

Substituting for y_2 , B_2 and D_1 using equations (1), (6) and (4) respectively, and performing the necessary algebra, we have

$$D_2 = \frac{x_1 - x_2}{x_2(\alpha_2 - 1)} - \frac{(x_0 - x_1)(x_1 - x_2)}{x_1 x_2 (\alpha_1 - 1)(\alpha_2 - 1)} \quad (7)$$

Now an expression for ϕ in terms of x_1 can be written. Using equations (1), (4) and (7), we have

$$\phi = \frac{(x_0 - x_1)\alpha_1}{(\alpha_1 - 1)} + \frac{\alpha_2(x_1 - x_2)}{(\alpha_2 - 1)} - \frac{(x_0 - x_1)(x_1 - x_2)\alpha_2}{x_1(\alpha_1 - 1)(\alpha_2 - 1)} \quad (8)$$

For a minimum we require that

$$\frac{\partial \phi}{\partial x_1} = 0$$

Performing the necessary differentiations, and collecting terms, we have

$$\frac{\alpha_1}{1 - \alpha_1} - \frac{\alpha_2}{1 - \alpha_2} - \frac{\alpha_2}{(1 - \alpha_1)(1 - \alpha_2)x_1^2} (x_0 x_2 - x_1^2) = 0$$

which implies

$$x_1 = \left[\frac{x_0 x_2}{(1 + \beta)} \right]^{1/2} \quad (9)$$

where

$$\beta = \frac{\alpha_1(1 - \alpha_2) - \alpha_2(1 - \alpha_1)}{\alpha_2} \quad (10)$$

We note that β vanishes if α_1 and α_2 are equal, in which case x_1

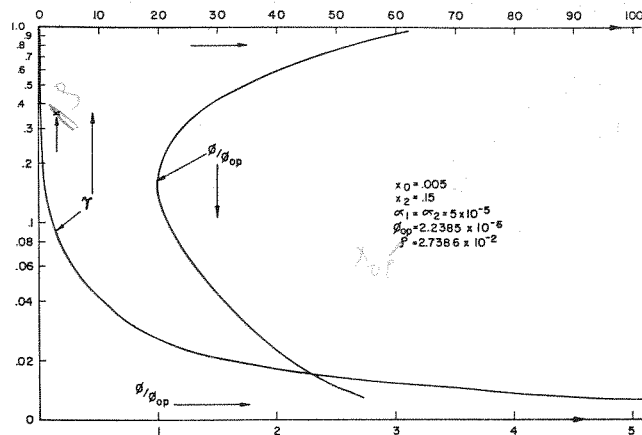


Fig. 2 Variation of solids carry-over with dimensionless intermediate concentration

becomes the geometric mean of x_0 and x_2 .

Finally, it is of some interest to examine the ratio of solute carry-overs in the two evaporators corresponding to the optimum condition, i.e., when x_1 satisfies equation (9).

Let

$$\gamma = \frac{D_2 y_2}{D_1 y_1} \quad (11)$$

Substituting for D_1 , D_2 , y_1 and y_2 as before, we have

$$\gamma = \frac{\alpha_2}{\alpha_1(1 - \alpha_2)} + \frac{\alpha_1 - \alpha_2 + \alpha_1 \alpha_2}{\alpha_1(1 - \alpha_2)} \frac{x_1}{x_1 - x_0} - \frac{\alpha_2}{1 - \alpha_2} \frac{x_2}{x_1 - x_0} \quad (12)$$

For the case of equal de-entrainment efficiencies, ($\alpha_1 = \alpha_2$), it can be shown that in the limit, as α_1 tend to zero, γ approaches unity.

In practical evaporators, the values of α are small, less than 10^{-4} . Therefore, we conclude that the minimum carry-over of solids to the distillate for this case occurs when the solids carried over from the two stages are equal.

3 Discussion

For purposes of illustration, let us consider the problem of concentrating a feed containing 0.5 percent total solutes to 15 percent concentration using a two stage evaporator. The de-entrainment factors for the two evaporator units are assumed to be equal to 0.5×10^{-4} . Thus we have $x_0 = 0.005$, $x_2 = 0.15$, and $\alpha_1 = \alpha_2 = 0.5 \times 10^{-4}$.

Equation (9) yields optimum intermediate concentration $x_1 = x_{op} = 2.7386 \times 10^{-2}$. The corresponding value of gross entrainment, $\phi = \phi_{op}$ is calculated using equation (8) to be 2.2385×10^{-6} . Fig. 2 shows ϕ/ϕ_{op} as a function of ρ where

Nomenclature

B_1 = discharge rate from Unit #1
 B_2 = concentrate (bottoms) discharge rate from Unit #2 (system discharge rate)
 D_1 = rate of vaporization (distillate) in Unit #1
 D_2 = rate of vaporization (distillate) in Unit #2
 x_0 = solute concentration in the feed

x_1 = solute concentration in bottoms in Unit #1, (intermediate system concentration)
 y_1 = entrainment concentration in the distillate from Unit #2
 y_2 = entrainment concentration in the distillate from Unit #2
 α_1, α_2 = de-entrainment factors for Unit #1 and 2, respectively

γ = ratio of solids in distillates of Unit #2 and #1
 ρ = progress function at intermediate concentration, dimensionless (equation 13)
 ϕ_{op} = total solute entrainment in the distillate under optimized conditions
 ϕ = total solute entrainment in the distillate (generic term)

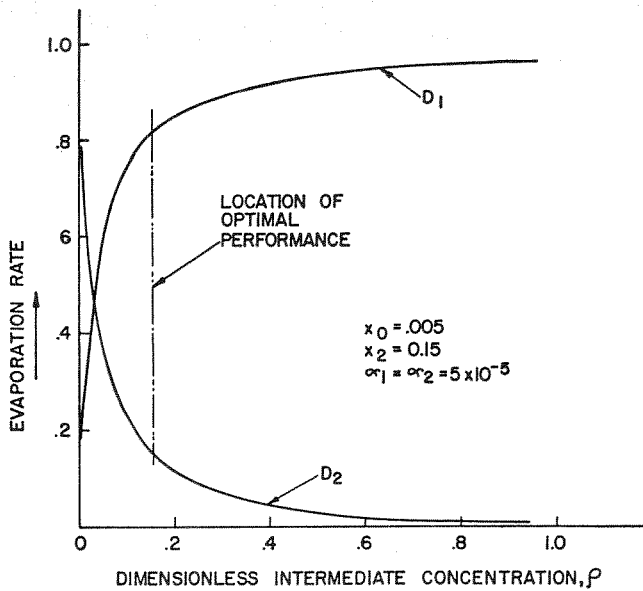


Fig. 3 Quantity of distillate in the two units versus intermediates

$$\rho = \frac{x_1 - x_0}{x_2 - x_0} \quad (13)$$

We note that ϕ decreases steeply as x_1 is increased and reaches a minimum at the optimal value of $x_1 = x_{OP}$. As x_1 is increased further, ϕ increases somewhat less steeply. The shape of this curve suggests operating the evaporator at an intermediate concentration which is equal to or "slightly" greater than x_{OP} . Operating below x_{OP} should be avoided. As x_1 approaches x_2 ; i.e. ρ approaches 1.0, the case of a single stage evaporator is simulated. It is instructive to note that ϕ/ϕ_{OP} is greater than 3 for $\rho = 1$. This means that the gross solute entrainment in the two stage evaporator will be less than one third of the amount of entrainment in an equivalent single stage evaporator. This example illustrates the effectiveness of a two stage system in reducing solute entrainment in the distillate.

Fig. 2 also shows γ as a function of ρ . γ is also shown to drop sharply with increasing ρ for $x_1 < x_{OP}$. It is approximately equal to one for $x_1 = x_{OP}$. The curve for γ is monotonic as would be expected from physical reasoning.

Fig. 3 shows D_1 and D_2 as functions of ρ . We note that D_1 is substantially larger than D_2 at the optimal point. This indicates that the equipment for the second stage will be much smaller than for the first stage. Furthermore, the curves for D_1 and D_2 are relatively steep for $x_1 < x_{OP}$, and flatten out for $x_1 > x_{OP}$. This fact is consistent with the earlier observation that the output quantities are mathematically strong functions of the intermediate concentration for small values of x_1 .

4 Conclusion

Equations to determine the optimum operating intermediate concentration in a two stage evaporator are derived. It is shown that the gross solute entrainment in a two stage evaporator operating at the optimal intermediate concentration is significantly less than a single stage system. Thus two stage systems merit careful consideration whenever stringent decontamination factors are required. It is also shown that the purest distillate is obtained when the carry-over from the stages are equal, if $\alpha_1 = \alpha_2$.

It is shown that the second stage is usually very small in size compared to the first stage. Thus, the hardware costs associated with the second stage will be quite small in comparison to the first stage. Hence, a small additional investment in equipment can yield significant gains in system performance. This, in a nutshell, is the main appeal of a two stage system in the Radioactive Waste Industry.

APPENDIX

Relation Between ϕ and ψ

The concentration of solids in the distillate is given by equation (2a).

$$\psi = \frac{\phi}{D_1 + D_2}$$

Using equation (6), we have

$$\psi = \frac{\phi}{1 - B_2} \quad (A1)$$

Solute mass balance in the system yields

$$x_0 = D_1 y_1 + D_2 y_2 + B_2 x_2$$

or

$$x_0 = \phi + B_2 x_2 \quad (A2)$$

Substituting for B_2 into equation (A1), we have

$$\psi = \frac{x_2 \phi}{x_2 - x_0 + \phi} \quad (A3)$$

Hence

$$\frac{\partial \psi}{\partial x_1} = \frac{x_2 (x_2 - x_0)}{(x_2 - x_0 + \phi)^2} \cdot \frac{\partial \phi}{\partial x_1} \quad (A4)$$

Since $x_2 > x_0$, and $\phi > 0$ by definition of the problem, the denominator in the r.h.s. of equation (A4) cannot vanish. Hence $\partial \psi / \partial x_1$ will be zero whenever $\partial \phi / \partial x_1$ is zero. In other words, the minima of ψ and ϕ functions coincide.