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On Minimization of Rad-Waste Carryover in an *n*-Stage Evaporator

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The mathematical problem of minimizing gross solids entrainment in an n*-stage evaporator is formulated and solved using the method of Lagrange multipliers. The solution procedure enables direct comparison of the decontamination efficiencies of multistage evaporators as the number of stages (*n*) is varied. A numerical example is utilized to illustrate the method of solution. Equivalent expressions for batch distillation are also derived.*

INTRODUCTION

Nuclear power plants produce large quantities of wastewater that contain small amounts of radioactive matter. Almost all the radioactive substance is attached to nonvolatile dissolved salts or undissolved solids in the water. Thus it is possible to decontaminate the wastewater through distillation. The technology for separation of solids from liquid solutions is quite old and well developed in the chemical and petrochemical industry. Industrial evaporators employed in the nuclear industry have drawn upon this well-established technology with satisfactory results [1]. The primary objective in the commercial distillation plants, however, is to obtain the "bottoms" of a certain concentration, the "bottoms" being the product.

Even in non-nuclear applications, the amount of carryover directly affects the fouling rate of heat transfer apparatus if the vapor is used as the heating medium in a subsequent stage. Excessive fouling derates the thermal performance of the heater and, in some exchanger constructions, jeopardizes its mechanical integrity as well [2].

Heightened concern about the environment is also a motivating factor to minimize the solids carryover in the vapor. Therefore, improving the purity of the distillate is a problem of general interest in all distillation applications. In the nuclear power plants, the problem is particularly acute since the contaminated water must be distilled to extremely high levels of purity.

Two approaches are available for improving the purity of the distillate. The first approach consists of using enhanced de-entrainment apparatus, since all nonvolatile solids carryover occurs through the entrained droplets of concentrate that are in the vapor. The second method for reducing net solids carryover is through multistage evaporation.

Holtz and Singh [3] have shown that the total entrained solids content of the distillate can be greatly reduced if the vaporization is done in two stages. There exists an optimal duty ratio between the two stages that leads to the minimum solids carryover. In [3], an analytical expression was

The computer program EVAP used in developing the numerical results is available from the authors at no charge.

obtained for the correct division of evaporative duty between the two stages. Using a numerical example, it was shown that a two-stage optimally proportioned evaporator yields a distillate that is over three times purer than a single-stage evaporator, if all evaporators are assumed to have identical de-entrainment efficiencies, and that optimum occurs when the concentration of the intermediate liquid is the geometric mean of the initial and final concentrations.

In this paper, we seek to study a generalized version of the problem treated in [3]. The question posed herein is: What is the optimal duty* arrangement for an n-state evaporator where n can be any integer? Furthermore, how does the total solids carryover change with n? We aim to formulate the governing equations in terms of arbitrary n such that a plot of ϕ_n (total solids carryover) with respect to n can be obtained. The designer now can select the optimal n by considering the hardware cost in conjunction with maintainability and reliability considerations.

The limiting case of multistage evaporation is batch distillation. Expressions of ϕ for batch distillation are also derived here. It is shown using a numerical example that the decontamination factor ϕ_b for batch distillation constitutes the upper bound on the maximum achievable purity in the distillate.

The method of Lagrange multipliers [4] is used to find the constrained optimum for an n-stage evaporator system. The solution procedure is described in the following section.

FORMULATION

Let us consider an n-stage evaporator (Fig. 1) employed to receive feed of solids concentration x_0, and deliver bottoms of concentration x_n. The feed rate can be assumed to be unity without any loss of generality. The quantity of the distillate produced in stage i is denoted by D_i. The concentration of the mother liquor in stage i is denoted by x_i, and the corresponding distillate is assumed to have solids concentration y_i. The object is to determine D_i such as to minimize the total solids carryover in the distillate ϕ_n. ϕ_n can be expressed as

$$\phi_n = \sum_{i=1}^{n} D_i y_i \tag{1}$$

It is reasonable to suppose that since the solids component in y_i originates as droplets of boiling liquid of concentration x_i, y_i will be proportional to x_i for a given de-entrainment apparatus:

$$y_i = \alpha_i x_i \tag{2}$$

where α_i denotes the de-entrainment efficiency of stage i.

Equation (2) characterizes the vaporization and de-entrainment properties of the hardware. The linear relationship utilized here is not due to a limitation in the technique of analysis presented here. It is used because of its ability to simulate real-life evaporators as well as its simplicity. Substituting Eq. (2) into Eq. (1) yields

$$\phi_n = \sum_{i=1}^{n} D_i \alpha_i x_i \tag{3}$$

In Eq. (3), ϕ_n contains n values of distillation rates D_i (one for each evaporator unit); and $(n-1)$ values of solids concentration x_i (one for each stage, except x_n which is the target concentration in the "bottoms"). This set of $(2n-1)$ variables

*Optimal duty arrangement is defined as one that leads to the least solids carryover in the distillate.

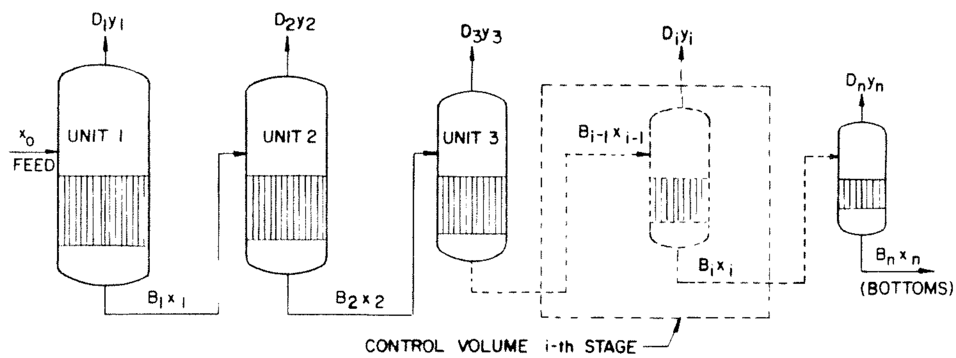

Figure 1 Flow diagram of solids in an n-stage evaporator.

can be adjusted to minimize ϕ_n. However, there are mass conservation requirements in the system that must be met. It is convenient to express mass continuity in terms of mass conservation of solids around each stage. We note that solids balance around the ith stage yields (Fig. 1):

$$B_{i-1}x_{i-1} = D_i y_i + B_i x_i \qquad i = 1, 2, \ldots, n \tag{4}$$

where B_i is the discharge rate from unit i.

Furthermore, the discharge rate from unit i is equal to the feed to the system less the quantity of distillate leaving the system in all units preceding unit i.

$$B_i = 1 - \sum_{j=1}^{i} D_j \qquad i = 1, 2, \ldots, n \tag{5}$$

From Eqs. (4), (5), and (2), we obtain

$$\left(1 - \sum_{j=1}^{i-1} D_j\right) x_{i-1} = D_i \alpha_i x_i + \left(1 - \sum_{j=1}^{i} D_j\right) x_i \tag{6}$$

or

$$x_{i-1}(1 + D_i) - x_i(1 + D_i \alpha_i) + (x_i - x_{i-1}) \sum_{j=1}^{i} D_j = 0 \qquad i = 1, 2, \ldots, n \tag{7}$$

In principle, Eq. (7) can be utilized to express all D_i in terms of x_i ($i = 0, 2, \ldots, n$). Substituting for D_i in Eq. (3) renders ϕ_n into a function of x_i ($i = 1, 2, \ldots, n-1$) alone. The minimum of ϕ_n can then be obtained by setting the partial derivative of ϕ_n with respect to x_i equal to zero. A more systematic procedure to obtain such a minimum is to consider ϕ_n as defined by Eq. (3) a function of $(2n - 1)$ unknowns, and treat the n algebraic equations in Eq. (7) as constraints to the optimization problem. The method of Lagrange multipliers [4] provides the ideal formalism to effect the solution. Accordingly, we form the functional ψ defined by

$$\psi = \sum_{j=1}^{n} D_j \alpha_j x_j + \sum_{i=1}^{n} \left\{ \lambda_i \left[x_{i-1}(1 + D_i) - x_i(1 + D_i \alpha_i) + (x_i - x_{i-1}) \sum_{j=1}^{i} D_j \right] \right\} \tag{8}$$

where λ_i ($i = 1, 2, \ldots, n$) are the undetermined multipliers. For minimum ϕ_n we require that

$$\frac{\partial \psi}{\partial D_m} = 0 \qquad m = 1, 2, \ldots, n \tag{9a}$$

$$\frac{\partial \psi}{\partial x_m} = 0 \qquad m = 1, 2, \ldots, n-1 \tag{9b}$$

Performing the necessary differentiations we have

$$(1 - \lambda_m)\alpha_m x_m + \lambda_m x_{m-1} + \sum_{i=m}^{n} \lambda_i (x_i - x_{i-1}) = 0 \qquad m = 1, 2, \ldots, n \tag{10a}$$

$$D_m \alpha_m (1 - \lambda_m) + (\lambda_{m+1} - \lambda_m)\left(1 - \sum_{j=1}^{m} D_j\right) = 0 \qquad m = 1, 2, \ldots, n-1 \tag{10b}$$

Equations (10a), (10b), and (7) furnish the required $(3n - 1)$ equations to solve for $(3n - 1)$ unknowns, namely x_i ($i = 1, 2, \ldots, n-1$); D_i and λ_i ($i = 1, 2, \ldots, n$). The solution of a set of simultaneous nonlinear algebraic equations can be found using a standard solution technique, such as Newton-Raphson. Since subroutines for such a task are now commonly available on most computer systems, we will not digress here into the details of the numerical analysis.

Before illustrating the application of the above formulation, we will derive an equivalent expression for ϕ_n for batch distillation, also known as Rayleigh distillation [5].

BATCH DISTILLATION

Let us assume that the initial quantity of liquor in the still is w_0 at (solids) concentration x_0. The final (required) concentration is x_n. At a typical point in the vaporization process, the solids concentration in the liquor is denoted by x, and the corresponding accumulated distillate quantity is denoted by Ω. The vapor evolving from a liquor of solids concentration x has entrained solids in concentration y. It is assumed that y is proportional to x; that is,

$$y = \alpha x \tag{11}$$

duction of $d\Omega$ quantity of distillate yields the owing solids mass balance equation:

$$= (w - d\Omega)(x + dx) + y\, d\Omega \qquad (12)$$

ere w is the quantity of liquor in the still at this nt in distillation. Noting that $d\Omega = -dw$ we 'e, after neglecting terms of higher order and rranging terms,

$$\frac{1}{1-\alpha}\frac{dx}{x} = \frac{dw}{w} \qquad (13)$$

egrating Eq. (13) we have

$$= cx^{-\beta} \qquad (14a)$$

ere c is the integration constant, and

$$= \frac{1}{1-\alpha} \qquad (14b)$$

e initial conditions are, at $x = x_0$, $w = w_0$. nce,

$$= w_0 x_0^{-\beta} \qquad (15)$$

d

$$= w_0 x_0^{\beta} x^{-\beta}$$

$$= -\beta w_0 x_0^{\beta} x^{-(\beta+1)}\, dx \qquad (16)$$

.e total solids carryover into the distillate ϕ_b is 'en by

$$= \int y\, d\Omega$$

$$= -\alpha \int x\, dw$$

$$= \alpha\beta w_0 x_0^{\beta} \int x^{-\beta}\, dx \qquad (17)$$

rforming the integration and simplifying, we have

$$= w_0 x_0 \left(1 - \frac{x_0}{x_n}\right)^{\alpha/(1-\alpha)} \qquad (18)$$

tting w_0 equal to unity gives the total solids .trainment corresponding to unit feed input.

XAMPLE

We will consider the numerical example of [3] herein the optimal two-stage system was found r the following input data:

$x_0 = 0.005$

$x_n = 0.15$

$\alpha_i = 0.5 \times 10^{-4} \qquad i = 1, 2, 3, \ldots, n$

The de-entrainment efficiencies for all stages are assumed to be equal, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_n$. For the purpose of comparing the relative de-entrainment efficacies of various stages, we define the relative distillate purity factor ϕ_n^* as the ratio of the total solids carryover in n-stage system to that in a single stage system, i.e.,

$$\phi_n^* = \frac{\phi_n}{\phi_1} \qquad (19)$$

Figure 2 shows ϕ_n by drawing a smooth curve through values of ϕ_n for discrete values of n. We notice that large gains in the reduction of solids carryover occur in going to two-stage from single-stage design. Proceeding to a three-stage configuration produces less spectacular gains. The improvement in the distillate purity becomes more marginal as the number of stages is increased even further. As a matter of fact, the gain from five- to six-stage design is too minuscule to be practical. This example indicates that the most meaningful design configurations lie in the two- to three-stage range.

Figure 3 shows the rate of evaporation D_i in an optimized n-stage unit in histogram form. Location 1 in the abscissa has eight vertical bars showing the values of D_i for eight different stage configurations

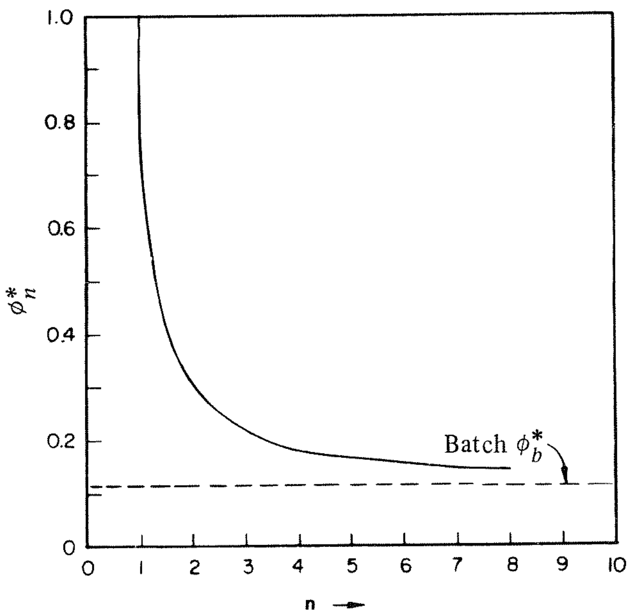

Figure 2 Variation of entrainment ratios with different stages of evaporator system.

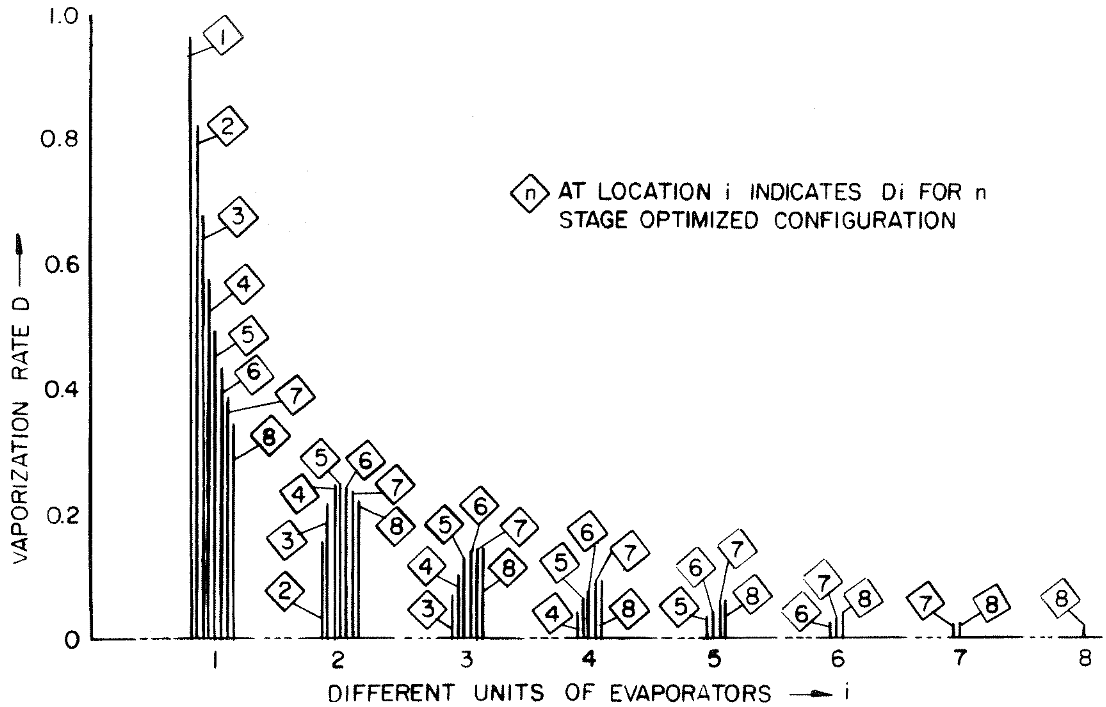

Figure 3 Variation of vaporization rate of each unit of evaporator in different stages of evaporation system.

(one-stage through eight-stage designs). It is to be noted that D_1 is much larger than D_i $(i > 1)$ for all optimized configurations. Since D_i directly determines the relative size of ith stage, it is apparent that the optimal designs require relatively small size equipment for all but the first stage. Thus, the additional cost of hardware in going from, for example, one-stage to two-stage, is small; the attendant improvement in purity is quite impressive.

Figure 2 also shows the value of ϕ_b for this problem. The horizontal line parallel to the abscissa indicates that the value of ϕ_b is the asymptote to the ϕ_n curve. This supports the intuitive result that batch distillation will produce the purest distillate. A multistage continuous evaporation system will yield a distillate whose purity depends on the number of stages. Indeed, the effectiveness of a multistage continuous distillation system can be measured in terms of its relative purity with respect to the corresponding batch distillation system. In the example problem, the two-stage optimized unit yields a distillate of 2.63 times higher solids concentration than the batch process. For a three-stage optimized system, the impurity factor is 1.85.

The numerical results will, of course, vary depending on the specifics of the problem. The overall trend of the results, however, does not change.

SUMMARY

The governing equations to characterize an n-stage continuous evaporator system are developed. The net solids carryover in an n-stage system can be minimized to determine the optimal ratio of evaporation duty in all the constituent stages and the corresponding liquor concentration. The well-known method of Lagrange multipliers is used to formulate this optimization problem. The solution of an n-stage system entails solving for $(n-1)$ x's, n D's, and n λ's. D_i $(i = 1, 2, \ldots, n)$ determines the evaporation equipment size for the ith stage. x_i $(i = 1, 2, \ldots, n-1)$ determines the corresponding liquir concentration. λ_i are the unknown multipliers. Thus $(3n-1)$ algebraic equations are solved for $(3n-1)$ unknowns.

The numerical solution for an example problem demonstrates the advantage of multistage vs. single-stage distillation. The purity of the distillate improves as the number of stages increases. However, more stages imply more instrumentation, piping, and equipment costs, and possibly reduced reliability, an engineering trade-off decision for each practical case is necessary. Two- and three-stage systems appear to be the most viable choices in most situations.

Expressions for the net solids carryover for batch distillation (Rayleigh distillation) are also derived for completeness. Batch process provides

the upper bound on the maximum achievable distillate purity, for a given entrainment factor.

NOMENCLATURE

D_i rate of distillate produced in stage i
B_i flow rate of liquor from the ith stage
n number of stages
w amount of liquor in the evaporator at a typical point in the vaporization process
w_0 quantity of liquor to be vaporized in a batch
x_0 solids concentration in the feed
x_i solids concentration in the liquor produced in the ith stage
y_i solids concentration in the distillate produced in the ith stage
α_i de-entrainment efficiency for ith stage
λ_i Lagrange multipliers
ϕ_b total solids carryover for batch distillation
ϕ_b^* ratio of total solids carryover in a batch distillation process to that in a single stage system ($\phi_b^* = \phi_b/\phi_1$)
ϕ_n total solids carryover in the distillate in an n-stage evaporator
ϕ_n^* ratio of total solids carryover in an n-stage system to that in a single-stage system ($\phi_n^* = \phi_n/\phi_1$)
Ω accumulated distillate in a batch evaporator at a typical point during vaporization

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