Int. J. Pres. Ves. & Piping **20** (1985) 127–138

Foundation Stresses under Supports of Freestanding Equipment Subjected to External Loads

K.P. Singh
President and CEO, Holtec International
Marlton, NJ 08053

&

I. Gottesman

&

A.I. Soler
Chief Technology Officer, Holtec International
Marlton, NJ 08053

ABSTRACT

A numerically efficient method to establish the pressure distribution underneath freestanding supports of a rectangular planform is developed. The supporting foundation is modeled as a 'linear spring bed', and the support base plate is assumed to simulate a rigid body. The support is assumed to be subject to a three-dimensional loading system. A numerical example is used to illustrate the solution procedure.

NOMENCLATURE

a, b	Length and width, respectively, of base plate.
c	Foundation pressure coefficient.

Int. J. Pres. Ves. & Piping 0308-0161/85/$03·30

$H(x)$	Heaviside unit step function.
k	Spring constant of foundation.
M_x, M_y	Applied external moments (Fig. 2).
$p(x, y)$	Pressures at a generic point (x, y).
V_i	Volume of tetrahedron i $(i = 1, 2, 3)$.
W	Vertical downward load (Fig. 2).
$x_c^{(i)}, y_c^{(i)}$	x and y coordinates, respectively, of centroid of tetrahedron i (Table 1).
x_0, y_0	Intersection of neutral axis with x and y axes, respectively (Fig. 2).
α	Rotation of foundation base plate.
θ	Angle between neutral axis and x axis.
ϕ	Potential energy of loaded foundation.

INTRODUCTION

Most supports for plant capital equipment, such as pressure vessels and heat exchangers, employ a rectangular planform base plate. The base plate is usually anchored to the foundation by a number of anchor bolts. The methods to size the base plate and anchor bolts to withstand externally applied loadings (e.g. earthquake, windload, reactions from interconnected piping, etc.) are now well established in the literature,[1] and are routinely used in the industrial design work.

However, the conventional practice of anchoring equipment has certain well-known drawbacks. For example, the constraint imposed by the supports produces thermal stresses in the vessel under operating conditions, and the fixity of the vessel produces high stresses in the interconnected pipings at the nozzle-to-vessel junction. Installing an anchored pressure vessel implies aligning the base plate anchor holes and all nozzle piping connections. Often, a piping nozzle flange junction has to be mechanically forced into alignment which greatly increases its potential for losing leak tightness. Anchored vessels are also looked upon with disfavor when they must be installed in a potentially radioactive environment. A prominent example in this category is the spent fuel storage rack used in light water reactor installations for storing irradiated nuclear fuel. These so-called storage racks must be designed to be installed and removed in a rapid manner to minimize radiation exposure to the plant personnel. Freestanding—completely unanchored—

installation is obviously the most suitable choice for the fuel storage racks. The practice of unanchored pressure vessel installation, while not very common, offers an effective way to mitigate thermal stresses in many cases, and therefore merits consideration.

The first step in the viability of such a concept in a given case requires the evaluation of the stability of the vessel under the externally applied loads. In this paper, we present the (previously unpublished) elements of the formulation necessary to carry out the vessel foundation response analysis.

MATHEMATICAL MODEL

A typical pressure vessel is subjected to a wide variety of external loads. The design procedure to compute the equilibrating foundation response is well documented in the literature, and therefore will not be repeated here. The aggregate effect of the external loadings on a base plate can be expressed in terms of six orthogonal reactions (three forces and three moments) (Fig. 1). Of these, the two overturning moments and the vertical load must be considered together to characterize the foundation response. For the purpose of this analysis, the following simplifying assumptions are made:

(i) The base plate is assumed to be of rectangular shape, and is assumed to be adequately buttressed to simulate a rigid platen.
(ii) The support foundation, typically of the reinforced concrete type, is modeled as a bed of linear elastic springs which can only support compression loads.

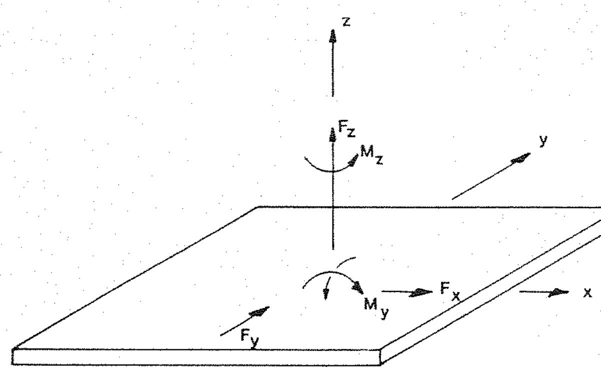

Fig. 1. Reaction loadings on the base plate.

Having idealized the problem in this manner, our object now is to determine the distribution of the interfacial pressure between the base plate and the concrete pedestal. Currently used design codes (Ref. 1, p. 939; Ref. 2) require that the peak surface pressure $p_{\max}$ be less than p_{limit}, where p_{limit} is given by:

$$p_{\text{limit}} = 0{\cdot}35 f_c'(A_2/A_1)^{1/2} \leq 0{\cdot}7 f_c' \tag{1}$$

where f_c' is the specified compressive strength of concrete (kips/in^2), A_1 is the bearing area (in^2) and A_2 is the full cross sectional area of concrete support (in^2).

It is noted that the value of p_{limit} depends on the extent of the loaded surface area underneath the base plate. The following analysis provides the method to compute both A_1 and $p_{\max}$.

ANALYSIS

Let a and b denote the length and width of the base plate, respectively (Fig. 2). The plate is subjected to vertical load W, and to overturning

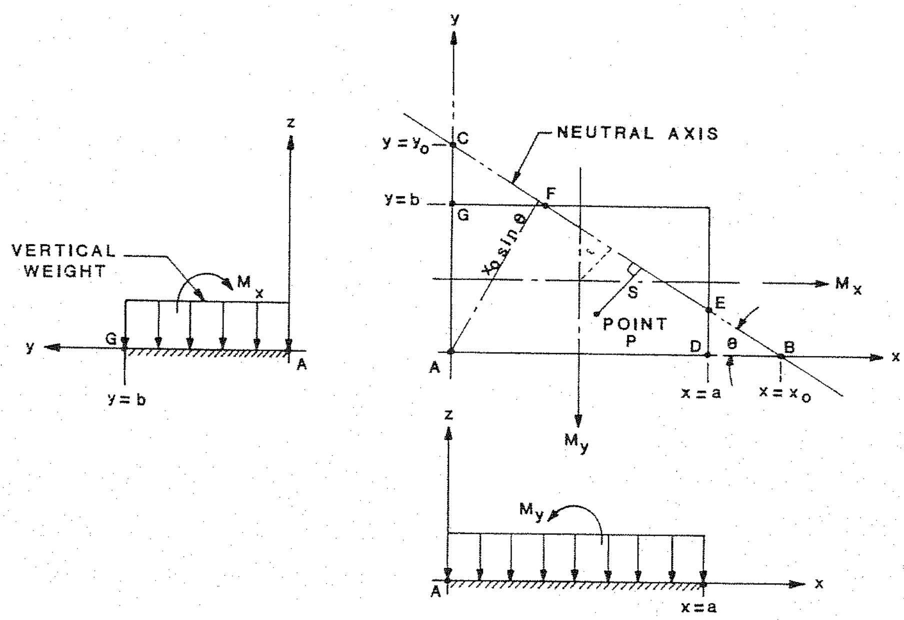

Fig. 2. Loading configuration on rectangular plate.

moments M_x and M_y acting along the global x and y axes, respectively. The 'linear spring bed assumption' for the concrete foundation implies that the pressure distribution on the plate–concrete interface is linear in the x and y coordinates. The loaded patch is characterized by a 'neutral axis', defined as the line along which the pressure is zero. In Fig. 2, line BC shows a typical neutral axis. The location of the neutral axis can be defined by a set of two coordinates, such as the coordinates of its intersection with the x and y axes (x_0 and y_0 in Fig. 2), or by x_0 and θ. The pressure at a point P located at distance s from the neutral axis is proportional to s.

As shown in Fig. 3, the pressure surface on the loaded region is therefore a tetrahedron of the general shape defined by the base quadrangle ADEFG and vertices H, I and J. The height of a point on the surface of the tetrahedron is equal to the distance of its projection on the xy plane from the neutral axis times the spring constant of the

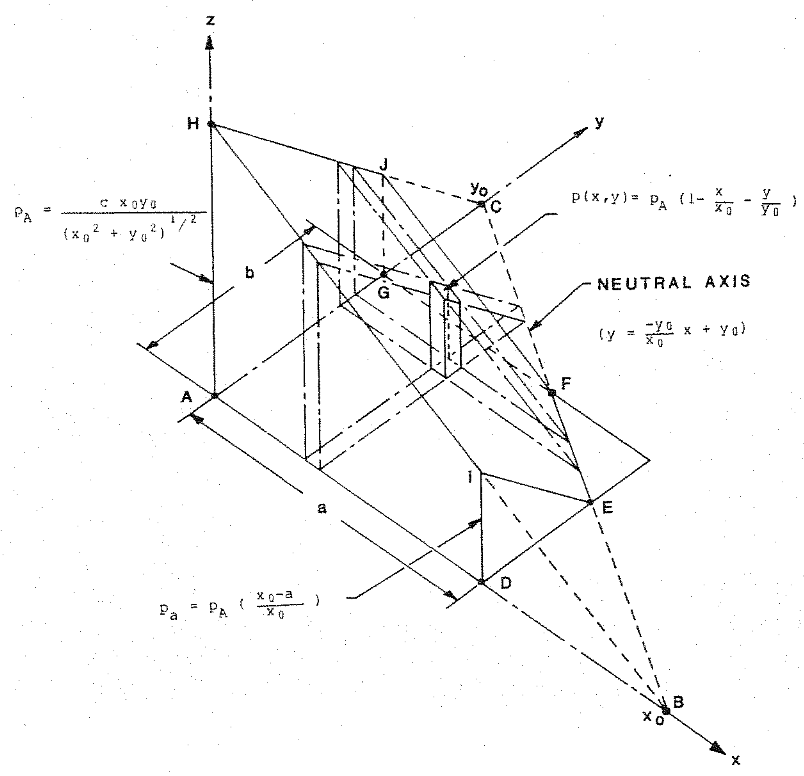

Fig. 3. Stress distribution envelope.

foundation, c. Thus, the height of point H in Fig. 3 is $cx_0 \sin\theta$. The expression for the 'pressure surface' can be found by using the standard equations of static equilibrium.

Vertical force equilibrium

The total vertical load W must equal the volume of the pressure tetrahedron ADEFGHIJ. We note that:*

$$[\mathrm{ADEFGHIJ}] = [\mathrm{ABCH}] - [\mathrm{BDEI}] - [\mathrm{CFGJ}] \tag{2}$$

The volume occupied by a tetrahedron is equal to one-sixth of the fourth order determinant defined by the vertices in the manner shown below.

$$[\mathrm{ABCH}] = V_1 = \frac{1}{6}\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x_0 & 0 & 0 \\ 1 & 0 & y_0 & 0 \\ 1 & 0 & 0 & cx_0 \sin\theta \end{vmatrix} \tag{3}$$

All terms in the first column are unity. The second, third and fourth columns are x, y and z coordinates of points A, B, C and H respectively. The quantity c is the 'spring coefficient' of the foundation, and $x_0 \sin\theta$ is the normal distance of point A from the neutral axis.

Multiplying out the determinant in eqn (3), we have

$$[\mathrm{ABCH}] = V_1 = \frac{c}{6} x_0^2 y_0 \sin\theta \tag{4}$$

or

$$[\mathrm{ABCH}] = V_1 = \frac{c}{6} x_0^3 \frac{(y_0/x_0)^2}{[1 + (y_0/x_0)^2]^{1/2}} \tag{5}$$

Similarly, the volume of the tetrahedron BDEI is given by

$$[\mathrm{BDEI}] = V_2 = \frac{H(x_0 - a)}{6}\begin{vmatrix} 1 & a & 0 & 0 \\ 1 & x_0 & 0 & 0 \\ 1 & a & (x_0 - a)\tan\theta & 0 \\ 1 & a & 0 & (x_0 - a)c \sin\theta \end{vmatrix}$$

$$= \frac{c}{6}(x_0 - a)^3 \sin\theta \tan\theta \, H(x - a) \tag{6}$$

* Square brackets are used to indicate the volume of the bounded region defined by the letters.

or

$$[\mathrm{BDEI}] = V_2 = \frac{c}{6}(x_0 - a)^3 \frac{(y_0/x_0)^2}{[1 + (y_0/x_0)^2]^{1/2}} H(x_0 - a) \tag{7}$$

where $H(x_0 - a)$ is the well-known Heaviside function defined as

$$\begin{aligned} H(x-a) &= 1 \qquad x > a \\ &= 0 \qquad x \leq a \end{aligned}$$

Similarly

$$[\mathrm{CFGJ}] = V_3 = \frac{H(y_0 - b)}{6} \begin{vmatrix} 1 & 0 & b & 0 \\ 1 & \dfrac{(y_0 - b)x_0}{y_0} & b & 0 \\ 1 & 0 & y_0 & 0 \\ 1 & 0 & b & \dfrac{cx_0(y_0 - b)}{y_0}\sin\theta \end{vmatrix}$$

or

$$[\mathrm{CFGJ}] = V_3 = \frac{H(y_0 - b)}{6} \frac{c(y_0 - b)^3}{(y_0/x_0)[1 + (y_0/x_0)^2]^{1/2}} \tag{8}$$

Substituting the expressions for the tetrahedron volumes into eqn (2) we have, after some algebra,

$$W = [\mathrm{ADEFGHIJ}] = \frac{c}{6} \frac{(y_0/x_0)^2}{[1 + (y_0/x_0)^2]^{1/2}} \times \left[x_0^3 - (x_0 - a)^3 H(x_0 - a) - \left(x_0 - \frac{bx_0}{y_0} \right)^3 H(y_0 - b) \right] \tag{9}$$

Equation (9) furnishes one relationship between x_0, y_0 and c.

Moment equilibrium

The coordinates of the centroids of the three tetrahedrons are also available from elementary coordinate geometry. These are reported in Table 1 for future reference.

Moment equilibrium about the x-axis yields

$$-\frac{b}{2} W + M_x = V_1 y_c^{(1)} - V_2 y_c^{(2)} - V_3 y_c^{(3)} \tag{10}$$

TABLE 1
Tetrahedron Centroids

	[ABCH]	[BDEF]	[CFGJ]
x-coordinate, x_c	$x_0/4$	$(x_0+3a)/4$	$(y_0-b)x_0/4y_0$
y-coordinate, y_c	$y_0/4$	$(x_0-a)(\tan\theta)/4$	$(y_0+3b)/4$
Volume	V_1	V_2	V_3
Tetrahedron number	1	2	3

where the superscript (i) indicates that the quantity pertains to tetrahedron i.

Substituting for V_1, V_2, V_3 from eqns (5), (7), (8) and for the centroidal coordinates from Table 1, and performing the necessary algebra, yields

$$\left(\frac{Wb}{2}-M_x\right)\left[1+\left(\frac{y_0}{x_0}\right)^2\right]^{1/2}-\frac{c}{24}\left(\frac{y_0}{x_0}\right)^3 \times\left[x_0^4-(x_0-a)^4H(x_0-a)-x_0^4\left(1-\frac{b}{y_0}\right)^3\left(1+\frac{3b}{y_0}\right)H(y_0-b)\right]=0 \tag{11}$$

Proceeding in an identical manner, the consequence of equilibrium about the y-axis is:

$$\left(\frac{Wa}{2}-M_y\right)\left[1+\left(\frac{y_0}{x_0}\right)^2\right]^{1/2}-\frac{c}{24}\left(\frac{y_0}{x_0}\right)^2 \times\left[x_0^4-(x_0-a)^3(x_0-3a)H(x_0-a)-\left(x_0-\frac{bx_0}{y_0}\right)^4H(y_0-b)\right]=0 \tag{12}$$

The three non-linear algebraic equations (9), (11) and (12) can be solved for c, x_0 and y_0 using a standard solution subroutine such as subroutine ZSCNT.[3] Unfortunately, there may be more than one plausible solution to the mathematical problem for certain geometry and loading conditions. It is necessary to identify the correct solution by some other means. The principle of minimum potential energy (Ref. 1, pp. 115, 212) is found to be an appropriate vehicle for this task.

Principle of minimum potential energy

The potential energy of the loaded foundation is given by:

$$\phi = U - \Gamma \tag{13}$$

where U is the total strain energy, and Γ is the sum of the product of the external loads and their respective movements. U and Γ can also be expressed in terms of x_0, y_0 and c as follows.

We note that the pressure surface in Fig. 3 is defined by

$$p(x,y) = p_A\left(1 - \frac{x}{x_0} - \frac{y}{y_0}\right) \tag{14}$$

Therefore the strain energy is given by

$$U = \frac{1}{2}\iint \frac{p(x,y)^2}{k}\,dx\,dy \tag{15}$$

where the integral extends over the hatched region in Fig. 4 and k is the foundation spring constant.

Substituting for $p(x, y)$ from eqn (14) into eqn (15) and integrating yields:

$$U = \frac{1}{24k}\frac{(cxy_0)^2}{(x_0^2+y_0^2)} \times \left[x_0y_0 - (x_0-a)^4\left(\frac{y_0}{x_0^3}\right)H(x_0-a) - (y_0-b)^4\left(\frac{x_0}{y_0^3}\right)H(y_0-b)\right] \tag{16}$$

where we have replaced p_A by its expression in terms of x_0 and y_0:

$$p_A = cx_0 \sin\theta = \frac{cx_0y_0}{(x_0^2+y_0^2)^{1/2}} \tag{17}$$

The quantity Γ in eqn (13) consists of the contributions from the applied loads W, M_x and M_y. The displacement of W can be shown to be

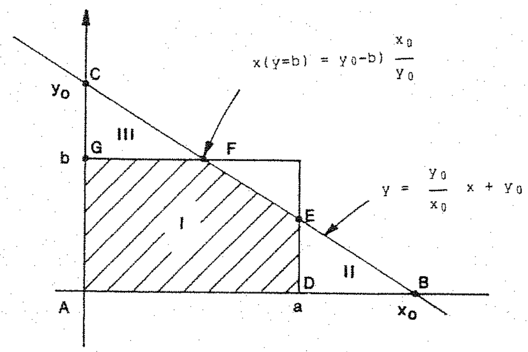

Fig. 4. Cross-section of the pressure surface on the foundation.

αl where $\alpha = c/k$ is the rotation of the baseplate, and l is shown in Fig. 2. Similarly, the rotations of M_x and M_y are given by $\alpha \cos\theta$ and $\alpha \sin\theta$ respectively.

Performing the required algebra, we have

$$\Gamma = \frac{cWx_0}{k(x_0^2+y_0^2)^{1/2}}\left[\frac{y_0}{x_0}\left(x_0-\frac{a}{2}\right)-\frac{b}{2}\right] + \frac{cM_x}{k}\frac{x_0}{(x_0^2+y_0^2)^{1/2}} + \frac{cM_y}{k}\frac{y_0}{(x_0^2+y_0^2)^{1/2}} \tag{18}$$

The potential energy ϕ is defined by eqns (13), (16) and (18) and is given as

$$\phi = \frac{1}{24k}\frac{(cx_0y_0)^2}{(x_0^2+y_0^2)^{1/2}} \times \left[x_0y_0-(x_0-a)^4\left(\frac{y_0}{x_0^3}\right)H(x_0-a)-(y_0-b)^4\left(\frac{x_0}{y_0^3}\right)H(y_0-b)\right] - \frac{c}{k(x_0^2+y_0^2)^{1/2}}\left\{Wx_0\left[\frac{y_0}{x_0}\left(x_0-\frac{a}{2}\right)-\frac{b}{2}\right]+M_xx_0+M_yy_0\right\} \tag{19}$$

or

$$k\phi = \frac{(cx_0y_0)^2}{24(x_0^2+y_0^2)^{1/2}} \times \left[x_0y_0-(x_0-a)^2\left(\frac{y_0}{x_0^3}\right)H(x_0-a)-(y_0-b)^3\left(\frac{x_0}{y_0^3}\right)H(y_0-b)\right] - \frac{c}{(x_0^2+y_0^2)^{1/2}}\left\{Wx_0\left[\frac{y_0}{x_0}\left(x_0-\frac{a}{2}\right)-\frac{b}{2}\right]+M_xx_0+M_yy_0\right\} \tag{20}$$

ϕ is computed for each plausible solution. The solution set which yields the minimum value of ϕ is the desired solution. We note that the value of k is not required, since k can be lumped with ϕ in the solution.

EXAMPLE

To illustrate the application of the solution method presented in the foregoing, we consider the following problem:

$a = b = 10$ in

case (i): $W = 100\,000$ lb, $M_x = M_y = 10^5$ lb-in
case (ii): $W = 80\,000$ lb, $M_x = M_y = 10^5$ lb-in
case (iii): $W = 150\,000$ lb, $M_x = M_y = 10^5$ lb-in
case (iv): $W = 100\,000$ lb, $M_x = 10^5$ lb-in, $M_y = 3 \times 10^5$ lb-in

Table 2 shows that, for two out of four cases, there are two plausible solutions. We have not found more than two 'candidate' solutions for any problem studied to date. The presence of possibly multiple solutions requires that the solution procedure search out all candidate output data in a systematic manner. The correct solution is sorted out by observing that ϕ must be an absolute minimum for that case.

TABLE 2
Results for the Example Problem

Loading case	*Solution 1*[a]				*Solution 2*			
	x_0 *(in)*	y_0 *(in)*	c *(lb/in³)*	$k\phi \times 10^{-8}$ *(lb/in)²*	x_0 *(in)*	y_0 *(in)*	c *(lb/in³)*	$k\phi \times 10^{-8}$ *(lb/in)²*
1	18·28	18·28	170·49	−0·62	29·23	29·23	78·91	−0·59
2	16·3	16·3	176·96	−0·44	—	—	—	—
3	22·8	22·8	165·18	−0·125	25·60	25·60	138·61	−0·124
4	7·40	21·15	643·17	−0·121	—	—	—	—

[a] Solution 1 is the correct solution since it has a lower value of the total potential energy.

CONCLUSION

The problem of the interaction of a reinforced concrete foundation to a freestanding support subject to externally applied loads has been formulated in a manner suitable for routine design work. The solution procedure is found to be numerically efficient, and suitable for running on a microcomputer. The loaded area of the support (also known as the 'bearing area') for a given set of external loads is calculated, along with the pressure distribution and locations of the neutral axis. In some codes[2] knowledge of the bearing area is necessary to determine the allowable surface pressure (see eqn (1)).

The computer program written for the problem is available from the authors, upon request.

REFERENCES

1. Singh, K. P. and Soler, A. I., *Mechanical Design of Heat Exchangers and Pressure Vessel Components*, Chapters 17 and 19, Arcturus Publishers, PO Box 606, Cherry Hill, NJ, 1984.
2. *Manual of Steel Construction*, Eighth edition, American Institute of Steel Construction, Chicago, 1980.
3. Scientific Subroutine Package, IMSL, Houston, Texas.