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Evaluation of AMPX-KENO Benchmark Calculations for High-Density Spent Fuel Storage Racks

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The AMPX-KENO computer code package is commonly used to evaluate criticality in high-density spent fuel storage rack designs. Consequently, it is important to know the reliability that can be placed on such calculations and whether or not the results are conservative. Recent critical experiments by the Babcock & Wilcox Company (B&W) provide data on configurations with thin absorbers containing boron that are more nearly representative of poisoned spent fuel storage rack designs than were earlier critical experiments.

A series of AMPX-KENO calculations has been made on selected critical experiments and the results compared with similar analyses reported in the literature by the Oak Ridge National Laboratory and B&W. Within the normal statistical variation of KENO calculations, results confirm that there is no apparent difference in the versions of AMPX-KENO and the 123-group GAM-THERMOS libraries used at three different computer installations.

Evaluation of the calculational results provides evidence for a statistically significant trend toward overprediction of reactivity with increasing reactivity worth of thin plates of boron-containing material. Similarly, statistical analyses reveal a trend toward underprediction of reactivity with increasing water-gap spacing between fuel assemblies. For most realistic spent fuel storage rack designs including neutron absorbers, these results imply that AMPX-KENO calculations are conservative and could possibly overpredict reactivity by as much as 2 to 5% Δk, based on a linear extrapolation of observed trends.

Statistical analyses in support of these contentions are provided, and additional critical experiments with boron absorbers of higher reactivity worth are recommended.

INTRODUCTION

Results of published AMPX-KENO calculations[1,2] of selected critical experiments pertinent to spent fuel storage rack benchmarking were compiled and some of the analyses repeated. The critical experiments included two series performed by Battelle-Pacific Northwest Laboratories[3,4] (BNWL) and one series performed by The Babcock & Wilcox Company[5,6] (B&W). The AMPX-KENO analyses of these

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[1] N. M. GREENE, J. L. LUCIUS, L. M. PETRIE, W. E. FORD, J. E. WHITE, and R. Q. WRIGHT, "AMPX: A Modular Code System for Generating Coupled Multigroup Neutron-Gamma Libraries from ENDF/B," ORNL-TM-3706, Oak Ridge National Laboratory (1976).

[2] L. M. PETRIE and N. F. CROSS, "KENO-IV, An Improved Monte Carlo Criticality Program," ORNL-4938, Oak Ridge National Laboratory (1975).

[3] S. R. BIERMAN et al., "Critical Separation Between Subcritical Clusters of 2.35 wt% U^{235} Enriched UO_2 Rods in Water with Fixed Neutron Poisons," PNL-2438, Battelle-Pacific Northwest Laboratories (1977); see also S. R. BIERMAN et al., *Nucl. Technol.*, **42**, 237 (1979).

[4] S. R. BIERMAN et al., "Critical Separation Between Subcritical Clusters of 4.29 wt% U^{235} Enriched UO_2 Rods in Water with Fixed Neutrons Poisons," NUREG/CR-0073, Battelle-Pacific Northwest Laboratories (1978), with errata sheet issued by the U.S. Nuclear Regulatory Commission on Aug. 14, 1979; see also S. R. BIERMAN et al., *Nucl. Technol.*, **42**, 237 (1979).

[5] M. N. BALDWIN et al., "Critical Experiments Supporting Close Proximity Water Storage of Power Reactor Fuel," BAW-1484-7, The Babcock & Wilcox Company (1979).

[6] G. S. HOOVLER et al., *Nucl. Technol.*, **51**, 217 (1980).

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experiments include the Oak Ridge National Laboratory (ORNL) analyses[7] and the B&W analyses.[5,6] Independent analyses were performed on four of the BNWL experiments and on the 21 experiments of the B&W series.

The significance of calculating the reactivity holddown by boron absorber plates can be emphasized by assuming, for illustrative purposes, a 10% error in calculating the reactivity worth of boron absorber plates. In the BNWL series of critical experiments, the resulting error would be ~0.003 Δk, which would be undetectable in the normal Monte Carlo statistical variation. For the B&W critical experiment of highest boron worth, a 10% error would result in an ~0.015 Δk deviation. However, for a realistic high-density spent fuel storage rack, the same 10% error in calculating the reactivity worth of boron absorber plates could result in an error of as much as 5% Δk. Thus, it is important to know whether or not the potential error in AMPX-KENO benchmark calculations is conservative.

COMPUTER FACILITIES AND OPERATING CODE PACKAGES

The principal computer facility used for the AMPX-KENO calculations was the PRIME system, operated by Ridihalgh, Eggers and Associates (REA) of Columbus, Ohio. Results of the calculations of a number of critical experiments were compared with corresponding values from calculations at the B&W CDC-7600 computer facility in Lynchburg, Virginia, and with the results reported by ORNL. Both the REA and the B&W versions of AMPX-KENO were unmodified copies of the ORNL code package and cross-section library distributed by the ORNL Radiation Shielding Information Center. Comparisons of calculational results confirm that there is no significant difference in the operating versions of the code package and libraries at the different computer installations.

CALCULATIONAL RESULTS

BNWL Critical Experiments

The BNWL has performed a series of critical experiments with thin plates of Boral absorber material. Independent analyses of these critical experiments have been performed with AMPX-KENO using the PRIME computer system (REA). Table I gives the results of these calculations, as well as corresponding calculational results reported by ORNL, with both sets of analyses using the 123-group GAM-THERMOS cross-section library. The agreement between the two sets of results confirms that there is no significant difference in the operating versions of AMPX-KENO or of the cross-section libraries at the two computer facilities.

TABLE I

Results of AMPX-KENO Calculations of BNWL Critical Experiments

Experiment Number	Reference	REA	ORNL (Ref. 7)
		k_{eff} σ^a	k_{eff} σ^a
16	3	1.005 ± 0.007	1.007 ± 0.005
17	3	0.997 ± 0.005	--- ---
20	3	1.015 ± 0.006	1.010 ± 0.004
31	4	0.996 ± 0.006	0.996 ± 0.005
Mean		1.003 ± 0.003	1.004 ± 0.003[b]
Variance		0.0000776	0.0000543
Standard deviation of mean		± 0.0044	± 0.0042

[a] One standard deviation.

[b] $\frac{1}{n}\left(\sum \sigma^2\right)^{1/2}$.

Analyses of the other critical experiments in the BNWL series, reported by ORNL, are given in Table II. Analysis of variance (*F*-test at 95% confidence limit) confirms that within each series, the dispersion in calculated k_{eff} values is within the normally expected statistical variation of KENO calculations. However, the mean k_{eff} for the first series is higher than that of the second series by a statistically significant amount. On the basis of the two series of critical experiments, it is not possible to uniquely define the reason for the observed difference, although it may be due to fuel density, lattice spacing, fuel enrichment, size of each fuel assembly, or to a combination of these factors.

To investigate the significance of the boron (Boral) absorber plate, the AMPX-KENO calculation for case 20 (see Table I) was repeated with the boron concentration set to zero. By difference between calculations with and without boron, the boron "worth" was estimated to be ~2.7% Δk. Because of the low reactivity worth of the boron absorber and the small core size (high neutron leakage), the BNWL series is believed to be less significant in benchmarking than the B&W series described below. Each of the three in-line fuel assemblies in the BNWL series is nearly critical alone, and the importance of the absorber plates between assemblies is therefore small. Similarly, the reactivity effect of water gaps between assemblies is too small to permit the reliable determination of a water-gap bias.

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[7] R. M. WESTFALL and J. R. KNIGHT, *Trans. Am. Nucl. Soc.*, **33**, 368 (1979).

TABLE II

ORNL Analyses of BNWL Critical Experiments

Series 1 2.35% ^{235}U-enriched UO_2 84% theoretical density 2.032-cm lattice pitch			Series 2 4.306% ^{235}U-enriched UO_2 94.9% theoretical density 2.54-cm lattice pitch		
Experiment Number	Plate Material	Calculated k_{eff}	Experiment Number	Plate Material	Calculated k_{eff}
15	---	1.008 ± 0.004	32	---	0.984 ± 0.004
5	---	1.005 ± 0.004	14	Type 304L stainless steel	0.989 ± 0.006
27	Type 304L stainless steel[a]	0.997 ± 0.004	13	Type 304L stainless steel	0.991 ± 0.005
26	Type 304L stainless steel	0.999 ± 0.004	8	Type 304L stainless steel	0.993 ± 0.005
28	Type 304L stainless steel	1.002 ± 0.004	7	Type 304L stainless steel	0.991 ± 0.005
29	Type 304L stainless steel	0.994 ± 0.004	10R	Type 304L stainless steel (1.05 boron)	0.987 ± 0.004
24	Type 6061 aluminum	0.998 ± 0.004	9	Type 304L stainless steel (1.05 boron)	0.986 ± 0.004
48	Type 6061 aluminum	0.997 ± 0.004	12	Type 304L stainless steel (1.62 boron)	0.985 ± 0.004
46	Zircaloy-4	0.996 ± 0.004	11	Type 304L stainless steel (1.62 boron)	0.992 ± 0.005
47	Zircaloy-4	0.998 ± 0.004	6	Type 6061 aluminum	0.983 ± 0.006
31	Copper	1.001 ± 0.005	5	Type 6061 aluminum	0.987 ± 0.005
12	Copper	1.000 ± 0.004	30	Zircaloy-4	0.984 ± 0.005
43	Copper	1.004 ± 0.004	29	Zircaloy-4	0.979 ± 0.005
44	Copper	1.007 ± 0.004	16	Copper	0.990 ± 0.005
41	Copper (0.989 cadmium)	1.014 ± 0.004	15	Copper	0.992 ± 0.005
36	Cadmium	1.005 ± 0.004	18	Copper	0.991 ± 0.004
37	Cadmium	1.006 ± 0.004	17	Copper	0.989 ± 0.004
50	Cadmium	0.998 ± 0.004	20	Copper (0.989 cadmium)	0.986 ± 0.005
54	Cadmium	1.007 ± 0.004	19	Copper (0.989 cadmium)	0.992 ± 0.004
52	Cadmium	1.009 ± 0.004	26	Cadmium	0.987 ± 0.005
20	Boral	1.010 ± 0.004	25	Cadmium	0.994 ± 0.004
16	Boral	1.007 ± 0.005	28	Cadmium	0.993 ± 0.004
32	Type 304L stainless steel (1.05 boron)	1.004 ± 0.004	27	Cadmium	0.985 ± 0.004
33	Type 304L stainless steel (1.05 boron)	0.999 ± 0.004	22	Cadmium	0.986 ± 0.004
38	Type 304L stainless steel (1.62 boron)	1.000 ± 0.004	21	Cadmium	0.995 ± 0.004
39	Type 304L stainless steel (1.62 boron)	1.002 ± 0.004	24	Cadmium	0.990 ± 0.005
			23	Cadmium	0.992 ± 0.005
			31	Boral	0.996 ± 0.005
Mean k_{eff}		1.003	Mean k_{eff}		0.989
Pooled variance		0.000017	Pooled variance		0.000022
Variance of means		0.000025	Variance of means		0.000017
Standard deviation of mean k_{eff}		±0.00098	Standard deviation of mean k_{eff}		±0.00077

Combined Population	
Mean k_{eff}	0.995
Pooled variance	0.000019
Variance of means	0.000068
Standard deviation of mean k_{eff}	±0.0011

[a] Type 304L stainless steel containing, in some instances, 1.05 or 1.62% boron.

B&W Critical Experiments

A series of 21 critical experiments was performed by B&W using a 3 X 3 array of fuel assemblies containing uranium enriched to 2.46 wt% in ^{235}U. Each assembly consisted of a 14 X 14 array of fuel rods (1.21-cm o.d., 1.030-cm pellet diameter) with an active height of ~5 ft. At a lattice pitch of 1.64 cm, the water-to-fuel volume ratio (1.88) is comparable to that (1.80 to 1.95) of current light water reactor (LWR) fuel assemblies. Therefore, except for the use of aluminum cladding rather than zirconium, the B&W series of critical experiments is reasonably representative of LWR spent fuel storage arrays. These critical experiments have been analyzed by AMPX-KENO, and the results are given in Table III, together with corresponding calculational results reported by B&W.

The close agreement between the mean, variance, and standard deviation of the mean of the two sets of analyses confirms that there is no significant difference in calculational results for the two computer installations.

TABLE III

Results of AMPX-KENO Calculations of B&W Critical Experiments

Experiment Number	REA	B&W	Boron Concentration in Moderator (ppm)
	k_{eff} σ^a	k_{eff} σ^a	
I	1.008 ± 0.006	0.998 ± 0.006	0
II	0.995 ± 0.004	1.007 ± 0.004	1037
III	1.009 ± 0.004	0.999 ± 0.004	764
IV	1.003 ± 0.005	1.004 ± 0.007	0
V	1.003 ± 0.004	1.005 ± 0.005	0
VI	1.002 ± 0.005[b]	0.988 ± 0.004[b]	0
VII	0.995 ± 0.004	0.994 ± 0.005	0
VIII	0.986 ± 0.005[b]	0.995 ± 0.005[b]	0
IX	0.984 ± 0.005[b]	0.984 ± 0.005[b]	0
X	0.986 ± 0.004	0.988 ± 0.004	143
XI	1.002 ± 0.004	1.015 ± 0.004	514
XII	0.998 ± 0.004	0.991 ± 0.005	217
XIII	1.008 ± 0.006[c] 1.011 ± 0.006 1.003 ± 0.005	1.008 ± 0.005	15
XIV	0.999 ± 0.004[d] 0.997 ± 0.004	1.003 ± 0.004	92
XV	0.996 ± 0.005	0.995 ± 0.005	395
XVI	0.988 ± 0.004	0.990 ± 0.005	121
XVII	0.997 ± 0.004	0.993 ± 0.005	487
XVIII	0.995 ± 0.005	1.005 ± 0.005	197
XIX	0.995 ± 0.003	0.991 ± 0.004	634
XX	0.994 ± 0.004	0.997 ± 0.005	320
XXI	0.983 ± 0.004	0.981 ± 0.004	72
Mean	0.996 ± 0.001[e]	0.997 ± 0.001[e]	
Pooled variance	0.0000185	0.000023	
Variance of k_{eff} values	0.0000597	0.0000751	
Standard deviation of mean k_{eff}	± 0.0017	± 0.0019	
Standard deviation of population of k_{eff} values	± 0.0077	± 0.0087	

[a] One standard deviation.

[b] Corrected for small deviation from criticality as reported by B&W.

[c] Mean of three calculations is 1.007 ± 0.003.

[d] Mean of two calculations is 0.998 ± 0.003.

[e] $\frac{1}{n}\left(\sum_i \sigma_i^2\right)^{1/2}$ expected standard deviation of mean k_{eff}.

The analyses of the 21 B&W critical experiments yielded a mean k_{eff} of 0.996 ± 0.0017, which is in good agreement with the corresponding mean k_{eff} of 0.996 ± 0.0011 derived from the ORNL calculations[7] with AMPX-KENO on 54 critical experiments. Combining the individual k_{eff} values for each generation in all the KENO calculations (REA) of the 21 B&W critical experiments into a single large population yields a grand mean of 0.996 ± 0.00096 (total sum of squares). However, examination of the reactivities calculated for the B&W criticals reveals that there is a considerable spread among the individual KENO values of k_{eff}. Analysis of variance techniques (F-test) confirms that, within a 95% confidence limit, the k_{eff} values for either set of analyses are not all from the same population, and the dispersion of calculated k_{eff} values is significantly larger than would be expected from the normal statistical variation of KENO calculations. Thus, there are apparently trends or factors other than statistical variations contributing to the observed deviations between calculated k_{eff} values, and the results cannot properly be treated as a single population without the possible risk of failing to account for a potentially significant factor. The two most important factors identifiable are (a) the effect of the size of the water gap between fuel assemblies, and (b) the effect of boron content in the absorber plates. Experimental uncertainties cited[5] for the critical experiment measurements are appreciably smaller than the statistical variations in the calculational results.

Effect of Water-Gap Spacing in B&W Critical Experiments

The B&W critical experiments were performed with different water gaps between fuel assemblies, ranging from no water gap to a spacing equivalent to four pin pitches (6.55 cm). The five subsets of critical experiment analyses, each with constant water-gap spacings, can be collected and analyzed independently, as indicated in Table IV. In each critical experiment, soluble boron of various concentrations (see Table III) was used to maintain a nearly constant critical height. However, no discernible trend with soluble boron concentration is readily apparent, despite the trend observed when boron is in thin absorber plates.

The reactivity data in Table IV clearly show a decrease in calculated k_{eff} with increasing water-gap spacing. Linear regression analysis of the mean k_{eff} values for each grouping in Table IV results in a Δk_{eff} of approximately −0.005 for each increase of one pin pitch in water-gap spacing (−0.0031 Δk/cm water gap above ~1.64 cm) over the range of experimental data.

Using this correlation, the calculated k_{eff} values for the B&W critical experiments can be corrected for bias due to water-gap spacing and normalized to a one-pin-pitch spacing (1.64 cm) as shown in Table V. The normalized k_{eff} values in Table V thus should be nearly independent of water-gap thickness. Analysis of variance (F-test) of the sample populations suggests that each of the two series of normalized k_{eff} values is approximately the same population, i.e., the variance ratio, F, of 1.58 (REA) and 2.03 (B&W) is not much larger than the expected F, (20, ∞, 5%) of 1.57. Although there may be a small additional perturbation in the corrected B&W analyses, the variation of k_{eff} values within each group of analyses is nearly within the expected statistical variation (within a 95% confidence limit). Furthermore, both the t-test and the F-test (95% confidence limits) confirm that the mean k_{eff} of the REA and B&W normalized values are equivalent, i.e., statistically the same populations.

TABLE IV

Subsets of Critical Experiment Analyses with Constant Water-Gap Spacings

Experiment Number	Calculated k_{eff} REA	Calculated k_{eff} B&W
Water Gap of No Pin Pitch		
I	1.008 ± 0.006	0.998 ± 0.006
II	0.995 ± 0.004	1.007 ± 0.004
Mean	1.002 ± 0.0036	1.003 ± 0.0036
Water Gap of One Pin Pitch		
III	1.009 ± 0.004	0.999 ± 0.004
IV	1.003 ± 0.005	1.004 ± 0.007
XI	1.002 ± 0.004	1.015 ± 0.004
XIII	1.007 ± 0.003	1.008 ± 0.005
XIV	0.999 ± 0.003	1.003 ± 0.004
XV	0.996 ± 0.005	0.995 ± 0.005
XVII	0.997 ± 0.004	0.993 ± 0.005
XIX	0.995 ± 0.003	0.991 ± 0.005
Mean	1.001 ± 0.0014[a]	1.001 ± 0.0018[a]
σ of Means	± 0.005	± 0.008
Water Gap of Two Pin Pitches		
V	1.003 ± 0.004	1.005 ± 0.005
VI	1.002 ± 0.005	0.988 ± 0.004
XII	0.998 ± 0.004	0.991 ± 0.005
XVI	0.988 ± 0.004	0.990 ± 0.005
XVIII	0.995 ± 0.005	1.005 ± 0.005
XX	0.994 ± 0.004	0.997 ± 0.005
Mean	0.997 ± 0.0018[a]	0.996 ± 0.0020[a]
σ of Means	± 0.0056	± 0.0076
Water Gap of Three Pin Pitches		
VII	0.995 ± 0.004	0.994 ± 0.005
VIII	0.986 ± 0.005	0.995 ± 0.005
X	0.986 ± 0.004	0.988 ± 0.004
XXI	0.983 ± 0.004	0.981 ± 0.004
Mean	0.988 ± 0.0021[a]	0.990 ± 0.0023[a]
σ of Means	± 0.0052	± 0.0065
Water Gap of Four Pin Pitches		
IX	0.984 ± 0.005	0.984 ± 0.005

[a] $\frac{1}{n}\left(\sum_i \sigma_i^2\right)^{1/2}$.

The mean k_{eff} of the 21 normalized values in Table V is 1.000 with a standard deviation of ±0.0012 (REA analyses). The maximum uncertainty is then ±0.003 Δk with a one-sided tolerance factor[8] of 2.371 (95% probability at a 95% confidence level for 21 samples). Thus, the calculational bias derived for AMPX-KENO is 0.000 ± 0.003 Δk, to which must be added a correction for water-gap thickness for specific fuel rack designs. Any uncertainty in the water-gap correction is inherently included in the level of uncertainty of the calculational bias. Since the water-gap correction would not normally be expected to be linear, extrapolation much beyond the range of measurement in the critical experiments is questionable.

Because the B&W critical experiments used single Boral plate absorbers (or rods) between fuel assemblies, application of the water-gap correction to storage rack designs using "flux-trap" configurations is uncertain. It seems reasonable to expect that water between two boron absorber plates will not contribute appreciably to the water-gap correction since this water will "see" thermally black absorbers on both sides. Thermal neutrons in the flux trap cannot escape, and consequently any error in thermal group cross sections of the water within the flux trap should not significantly affect reactivity. Thus, it is believed that the water-gap correction to the calculational bias should be based on the water outside the flux trap, between the boron-containing absorber plate and the fuel assembly, provided the absorber plates are essentially black to thermal neutrons.

Effect of Boron Content of Absorber Plates

Among the B&W critical experiments is a series of five measurements at a constant water gap using

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[8] M. G. NATRELLA, *Experimental Statistics*, U.S. National Bureau of Standards Handbook 91 (Aug. 1963).

TABLE V

Calculated k_{eff} Values for B&W Critical Experiments Normalized for Water-Gap Spacing

Experiment Number	Gap Thickness[a]	Δk Correction	k_{eff} Normalized to a Water Gap of 1.64 cm	
			REA	B&W
I	0	−0.005	1.003 ± 0.006	0.993 ± 0.006
II	0	−0.005	0.990 ± 0.004	1.002 ± 0.004
III	1	0	1.009 ± 0.004	0.999 ± 0.004
IV	1	0	1.003 ± 0.005	1.004 ± 0.007
V	2	+0.005	1.008 ± 0.004	1.010 ± 0.005
VI	2	+0.005	1.007 ± 0.005	0.993 ± 0.004
VII	3	+0.01	1.005 ± 0.004	1.004 ± 0.005
VIII	3	+0.01	0.996 ± 0.005	1.005 ± 0.005
IX	4	+0.015	0.999 ± 0.005	0.999 ± 0.005
X	3	+0.01	0.996 ± 0.004	0.998 ± 0.004
XI	1	0	1.002 ± 0.004	1.015 ± 0.004
XII	2	+0.005	1.003 ± 0.004	0.996 ± 0.005
XIII	1	0	1.007 ± 0.003	1.008 ± 0.005
XIV	1	0	0.998 ± 0.003	1.003 ± 0.004
XV	1	0	0.996 ± 0.005	0.995 ± 0.005
XVI	2	+0.005	0.993 ± 0.004	0.995 ± 0.005
XVII	1	0	0.997 ± 0.004	0.993 ± 0.005
XVIII	2	+0.005	1.000 ± 0.005	1.010 ± 0.005
XIX	1	0	0.995 ± 0.003	0.991 ± 0.004
XX	2	+0.005	0.999 ± 0.004	1.002 ± 0.005
XXI	3	+0.010	0.993 ± 0.004	0.991 ± 0.004
Mean			1.000 ± 0.001[b]	1.000 ± 0.001[b]
Pooled variance			0.0000185	0.000023
Variance of k_{eff} values			0.000029	0.000047
Standard deviation of mean k_{eff}			± 0.0012	± 0.0015
Standard deviation of population			± 0.0054	± 0.0069

[a] Gap thickness in terms of number of pin pitches (1.64 cm).

[b] $\frac{1}{n}\left(\sum_i \sigma_i^2\right)^{1/2}$, expected standard deviation of mean k_{eff}.

Boral absorber plates of differing boron content. These five critical experiments are identified in Table VI, which also gives results of the calculations for these experiments. Although some of the other critical experiments used boron absorbers between fuel assemblies, those measurements involved different water-gap spacings or cylindrical absorber rods and are not readily amenable to trend analysis.

The REA analyses in Table VI were each made with a different "seed," i.e., sequence of random numbers in KENO. However, the B&W calculations were made with the same seed, and the variance ratio of the B&W analytical results suggests a dependence on the boron content of the absorber plate. Linear regression analysis of the B&W calculated values (see Fig. 1) yields the following correlation:

$$k_{eff} = 0.9903 + 0.01072\,B \quad \text{(where B is the wt\% natural boron)}\ , \qquad (1)$$

with a correlation coefficient of 0.9965 and a standard error of estimate (root-mean-square about regression) of ±0.00059. The correlation and the 95% confidence intervals of the regression are shown graphically in Fig. 1. Using the same seed in the KENO calculations would be expected to have only a relatively unimportant effect on the statistical variations within the set of calculations. Nevertheless, the goodness-of-fit and the standard error of estimate lend credence to the validity of a trend in calculated k_{eff} with boron loading. In the REA analyses, the trend is not as clearly identifiable, but is consistent with that observed for the B&W analyses. As further

TABLE VI

Critical Experiments with Boral Absorber Plates

Experiment Number	Boron Content		AMPX-KENO Calculated k_{eff}	
	Boron[a] (wt%)	^{10}B Areal Density (g/cm^2)	REA	B&W
XIII	1.614	0.005	1.007 ± 0.003[b]	1.008 ± 0.005
XIV	1.257	0.004	0.998 ± 0.003[c]	1.003 ± 0.004
XV	0.401	0.0013	0.996 ± 0.005	0.995 ± 0.005
XVII	0.242	0.0008	0.997 ± 0.004	0.993 ± 0.005
XIX	0.100	0.0003	0.995 ± 0.003	0.991 ± 0.004
Mean			0.999 ± 0.002	0.998 ± 0.002
Pooled variance			0.000014	0.000021
Variance of k_{eff} values			0.000023	0.000052
Variance ratio[d]			1.71	2.43

[a] Weight percent natural boron in Boral plate of 2.7 g/cm^3 overall density and 0.645 cm thickness.

[b] Mean k_{eff} of three calculations.

[c] Mean k_{eff} of two calculations.

[d] Expected value $F(4, 228, 5\%) = 2.37$.

evidence of a trend in calculated k_{eff} with boron loading, calculations of the five critical experiments with Boral absorber were repeated (REA computer facility) with approximately twice the number of neutron histories. Results of these calculations, shown in Table VII, yield the following regression correlation:

$$k_{\text{eff}} = 0.9929 + 0.0104\,\text{B} \ , \tag{2}$$

which agrees well with the earlier correlation and tends to confirm the observed trend.

The five duplicate calculations with different random numbers (equivalent of 360 generations total) for critical experiment XIII (1.614 wt% boron) result in a mean of 1.009 and a weighted standard deviation of ±0.002. For a one-sided tolerance factor[8] of 1.78 (95% probability at a 95% confidence level with 360 generations), the expected mean is 1.009 ± 0.004 Δk. The reactivity "worth" of this level of boron in the B&W critical experiment was estimated to be 15.9% Δk, based on the difference in k_{eff} from two AMPX-KENO calculations, with and without boron. Thus, for a boron reactivity worth of ~16% Δk, AMPX-KENO overpredicts k_{eff} by 0.009 ± 0.004 Δk and, by extrapolation, would increasingly overpredict reactivity with increasing reactivity worth of the boron absorber. In most high-density fuel racks, the boron worth is in the range of 30 to 50% Δk. In this range, linear extrapolation of the regression analysis (on the basis of reactivity worth, not necessarily the boron absorber loading for any given configuration) suggests that AMPX-KENO could possibly overpredict k_{eff} by as much as 2 to 5% Δk. Although extrapolation of the regression trend much beyond the range of the measurements is questionable, the correlation does confirm that the AMPX-KENO calculations would be expected to conservatively overpredict k_{eff} when strong boron absorbers of high reactivity worth are present.

TABLE VII

Replicate Analyses of Critical Experiments with Boral Absorbers

Experiment Number	Natural Boron Content (wt%)	Calculated k_{eff} (REA)
XIII	1.614	1.014 ± 0.003
XIV	1.257	1.001 ± 0.003
XV	0.401	0.995 ± 0.003
XVII	0.242	0.994 ± 0.003
XIX	0.100	0.998 ± 0.003
Mean		1.000 ± 0.003
Pooled variance		0.000009
Variance of k_{eff} values		0.000065
Variance ratio		7.2

CONCLUSIONS

Of the critical experiments that have been published, the series performed by B&W is believed to be the most representative of poisoned spent fuel

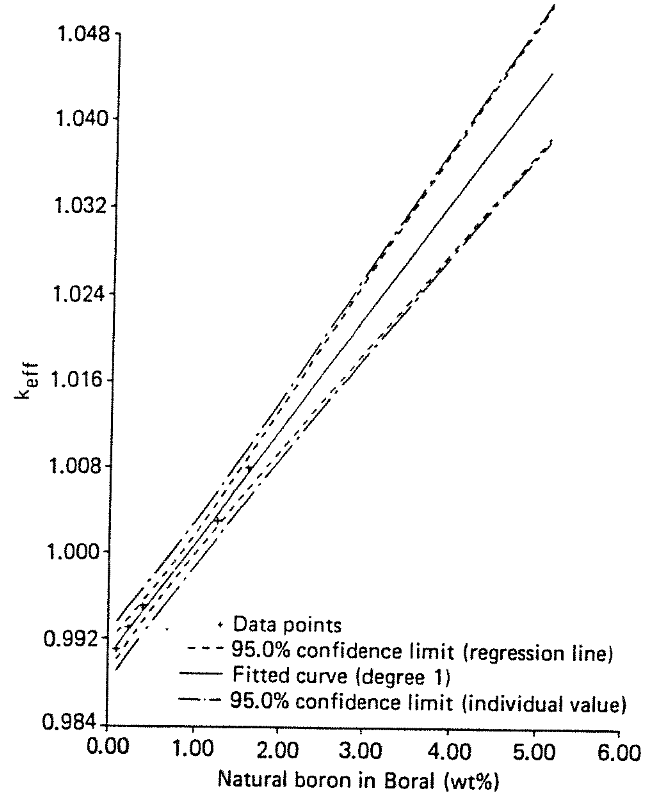

Fig. 1. Calculated k_{eff} versus natural boron content of Boral absorber plates.

storage racks. In the present analysis, AMPX-KENO calculations have been found to increasingly overpredict reactivity at higher reactivity worth of the boron absorber plates between fuel assemblies. Thus, AMPX-KENO would be expected to give conservative results in analyzing spent fuel storage racks. However, additional critical experiments, in the higher range of absorber worth, are needed to confirm the projected reactivity trend and to provide a reliable basis for adjusting the calculational method. In particular, critical experiments with flux-trap absorbers of reasonable reactivity worth are needed.

Based on the analyses and statistical trend evaluations given above, the following conclusions can be made.

1. Within the accuracy of the calculations, the versions of AMPX-KENO and the reference 123-group GAM-THERMOS library used by REA, B&W, and ORNL are all the same.

2. Analysis of 21 B&W critical experiments yields a mean k_{eff} of 0.996 ± 0.004 (95% probability and 95% confidence level). However, the dispersion of the population of calculated k_{eff} values (±0.018, 95% probability, and 95% confidence level) and the variance ratio indicate the benchmark k_{eff} values cannot properly be treated as a single population.

3. Trend analysis suggests that water-gap spacing and boron absorber worth are the principal factors responsible for the observed dispersion in the benchmark calculational results. Normalizing to a constant water-gap spacing (1.64 cm) results in a mean calculated k_{eff} of 1.000 ± 0.003 (95% probability and 95% confidence level) for the 21 B&W critical experiments, corresponding to a calculational bias of 0.000 ± 0.003.

4. With increasing size of water gap between fuel assemblies, AMPX-KENO tends to underpredict k_{eff}, and a correction factor of $-0.0031\ \Delta k$/cm water gap is necessary above a gap of 1.64 cm.

5. In general, a trend toward higher calculated k_{eff} values with increasing absorption in boron absorber plates was observed. Regression analysis indicates AMPX-KENO calculations will likely overpredict k_{eff} above a boron reactivity worth of ~10 to 15% Δk and confirms that, for most realistic spent fuel storage rack designs, AMPX-KENO will yield conservative values of k_{eff} when boron absorbers of higher reactivity worth are present.

6. For flux-trap absorber configurations, it is believed appropriate to exclude the water within the flux trap in developing a water-gap correction for a specific spent fuel storage rack design, provided the boron absorber plates are essentially "black" to thermal neutrons.

7. Additional critical experiments with boron absorbers of higher reactivity worth and with flux-trap configurations are needed to verify the trends observed here and to provide a basis for improving the accuracy of analytical methods.

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