Design of skirt-mounted supports

Skirt supported pressure vessels are popular because of minimum stresses at the vessel-skirt junction. Here are the equations and a computer program for a complete design analysis

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COLUMNS and heat exchangers are often supported on skirts. Skirt support is attractive from a designer's standpoint because it produces a minimum amount of local stresses caused by mechanical loads at its junction with the vessel. As a result, skirt support design is far more amenable to analysis than the lug type construction. Typically, the skirt support design must consider the following types of loadings:

- Dead weight of the equipment
- Operating weight
- Lateral loads by restrained thermal growth of interconnecting piping
- Wind loads
- Seismic loads.

Integrity of the foundation concrete, hold-down bolts and support plate for a whole spectrum of possible loading conditions must be investigated during design. Since the foundation stresses are a highly nonlinear function of the applied loadings, such an analysis may be very difficult without the aid of a computer. This article describes a procedure which has been successfully applied to solve technically important problems of this type. Equations to determine the stresses in the base plate or conversely, the required base plate thickness for a given allowable stress value are also given. The use of the method described here will result in significant economies of design in a majority of applications.

Method of solution. The loading applied on the equipment, regardless of their origin, result in four types of forces on the foundation, namely:

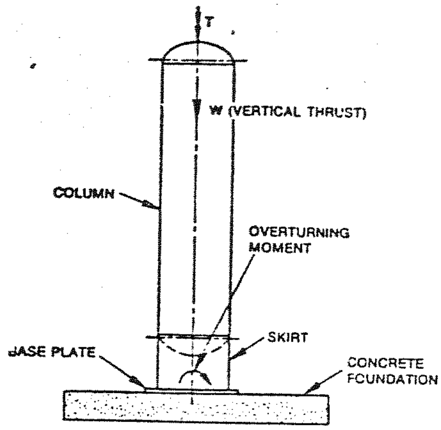

Fig. 1—Vertically mounted equipment under vertical load and overturning moment.

1. Vertical thrust W (positive if downwards) (Fig. 1)
2. Overturning moment M (Fig. 1) (shown vectorially in Fig. 2)
3. Axial twist T (mostly from piping reactions, Fig. 1)
4. Lateral shear, R (Fig. 2).

The magnitudes of W, M, T and R are found from the applied loadings by simple analysis.

The lateral shear force R makes a known angle Θ with the x axis. A solution for the maximum interfacial concrete pressure p and maximum bolt tensile stresses, σ, when subject to the combined action of axial thrust W and overturning moment M is developed as follows:

Axial thrust W and overturning moment M. Depending on the relative magnitudes of W and M, either of the following two conditions will exist:

No uplift. The concrete is in compression over the entire base plate region. This case will correspond to a large W and a relatively small M.

Nominal uplift. Part of the base plate (in the negative x-quadrants in Fig. 2) will lift up, resulting in a neutral axis located within the base.

The equations relating the peak concrete pressure p to

TABLE 1—Computer calculations converging to specified tolerance

Iteration # i	$p_i \times 10^{-3}$ (Eq. 19)	$\sigma_i \times 10^{-3}$ (Eq. 19)	a (Eq. 18)	φ_1 (Eq. 14)	φ_2 (Eq. 15)	φ_3 (Eq. 16)	φ_4 (Eq. 17)	$p_o' \times 10^{-3}$ (Eq. 12)	$\sigma_o' \times 10^{-3}$ (Eq. 13)
1	0.852	18.130	1.397	1.797	2.161	0.785	0.413	0.682	15.964
2	0.767	17.047	1.375	1.772	2.194	0.784	0.415	0.671	15.766
3	0.719	16.407	1.362	1.756	2.217	0.784	0.416	0.664	15.650
4	0.691	16.028	1.356	1.747	2.231	0.784	0.417	0.660	15.582
5	0.675	15.805	1.350	1.742	2.240	0.784	0.418	0.657	15.543
6	0.666	15.674	1.348	1.739	2.244	0.784	0.418	0.656	15.520
7	0.661	15.597	1.346	1.737	2.247	0.784	0.418	0.655	15.507
8	0.658	15.552	1.345	1.736	2.249	0.784	0.418	0.654	15.499
9	0.656	15.525	1.345	1.735	2.250	0.784	0.418	0.654	15.494

the applied M and W are found for the two conditions:

No uplift. Assuming that the foundation is linearly elastic, the linearly varying concrete pressure p_m is obtained as a function of the applied M by suming moments about the diametrical axis (y-axis in Fig. 2).

$$M = \frac{4\,p_m}{r_o}\left[\int_0^{r_o} x^2\,(r_o^2 - x^2)^{1/2}\,dx - \int_0^{r_i} x^2\,(r_i^2 - x^2)^{1/2}\,dx\right] \qquad (1)$$

where

r_o = Base plate outer radius
r_i = Base plate inner radius
M = Applied overturning moment.

It can be shown that; in general[5]

$$\int x^2\,(r^2 - x^2)^{1/2}\,dx = \frac{-x\,(r^2 - x^2)^{3/2}}{4} + \frac{x r^2\,(r^2 - x^2)^{1/2}}{8} + \frac{r^4\,\mathrm{Sin}^{-1}\frac{x}{r}}{8} \qquad (2)$$

using Equations 1 and 2, the following expression for the applied moment is found:

$$M = \frac{\pi\,(r_o^4 - r_i^4)\,p_m}{4 r_o} \qquad (3)$$

The maximum concrete pressure p is given as

$$p = p_m + p_o \qquad (4)$$

where the constant pressure term p_o is defined as

$$p_o = \frac{W}{\pi\,(r_o^2 - r_i^2)} \qquad (5)$$

and p_m is calculated from Equation 3. It is obvious that the bolts carry no stress in this case.

Most commonly, the width of the base plate, b, is small compared to its radii; i.e.

$$b = (r_o - r_i) << \tfrac{1}{2}\,(r_o + r_i) \qquad (6)$$

In practice, Relation 6 may be reduced to

$$b/(r_o + r_i) < 0.2 \qquad (6.a)$$

Thus, Equations 3 and 5 may be further simplified.

Let

$$s = (r_o + r_i) \qquad (7)$$

Then

$$r_o = 0.5s + 0.5b$$
$$r_i = 0.5s - 0.5b$$

Substituting Equation 7 in Equation 3 and Equation 5, and neglecting terms of higher order; we have

$$M = \frac{\pi s^2 b p_m}{4} \qquad (8)$$

$$p_o = \frac{W}{\pi s b} \qquad (9)$$

Equations 8 and 9 may be used to compute p_m and p_o instead of Equation 3 and 5, respectively, whenever Relationship 6 is valid.

The maximum concrete pressure p is obtained using Equation 4.

Finally, "no uplift" requires that

$$p_o \geq p_m \qquad (10)$$

If Relationship 10 is not satisfied then nominal uplift exists, for which the solution is obtained as follows:

Nominal uplift. A neutral axis parallel to the y-axis exists (see Fig. 2). The location of the neutral axis is identified by the angle α.

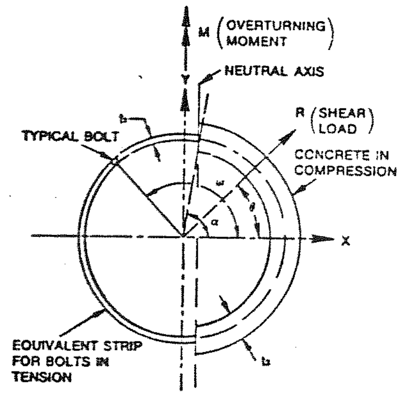

Fig. 2—Pressure distribution under base plate.

The object is to determine the maximum concrete pressure p and the angle α. The difficulty in obtaining the solution hinges on Relationship 6. If geometry of the base plate is such that the bound (6.a) is violated, then an exact solution (equivalent to Equations 3, 4 and 5 for the complete compression case) must be found, which is a rather cumbersome procedure. Since this situation is rather infrequent in practice, the solution for this case is not discussed further.

On the other hand, if Relationship 6 can be successfully invoked, then an approximate solution may be constructed with relative ease. It is assumed that the concrete annulus under the base plate may be treated as a thin ring of mean diameter c. Assuming the foundation to the linearly elastic, and the base plate to be relatively rigid, Brownell & Young[1] have developed an approximate solution which, with some modification, is summarized here and cast in a form suitable for a numerical solution.

Within the limits of accuracy sought, it is permissible to replace the bolts by an equivalent thin shell of thickness t of mean diameter equal to the bolt circle diameter c, such that

$$t = \frac{n_b A}{\pi c} \qquad (11)$$

Where the tensile stress area A of a bolt of nominal diameter d and n' threads per inch is given by[2]

$$A = \pi/4\left(d - \frac{0.9743}{n'}\right)^2$$

and there are n_b bolts. Also note that the bolt circle diameter is assumed equal to the diameter of the concrete annulus under the base plate.

Let n be the ratio of Young's moduli of the bolt material and the concrete (n normally varies between 10 to 15). Assuming that concrete can take only compression and bolts are effective only in tension (untapped holes in base plate), an analysis similar to the "no uplift" case above gives the following results:

$$\therefore \quad p = \frac{2W + \rho_2 t c \sigma}{\rho_1 c b} \tag{12}$$

$$\sigma = \frac{2(M - W\rho_4 c)}{\rho_2 \rho_3 t c^2} \tag{13}$$

where the dimensionless quantities ρ_1, ρ_2, ρ_3 and ρ_4 are defined as follows:

$$\rho_1 = \frac{2(\text{Sin}\,\alpha - \alpha\,\text{Cos}\,\alpha)}{(1 - \text{Cos}\,\alpha)} \tag{14}$$

$$\rho_2 = \frac{2}{1 + \text{Cos}\,\alpha}\left[(\pi - \alpha)\,\text{Cos}\,\alpha + \text{Sin}\,\alpha\right] \tag{15}$$

$$\rho_3 = \frac{1}{2}\left[\frac{(\pi - \alpha)\,\text{Cos}^2\,\alpha + \frac{\pi - \alpha}{2} + \frac{3\,\text{Sin}\,2\alpha}{4}}{(\pi - \alpha)\,\text{Cos}\,\alpha + \text{Sin}\,\alpha}\right] + \frac{1}{2}\left[\frac{\frac{\alpha}{2} + \alpha\,\text{Cos}^2\,\alpha - \frac{3\,\text{Sin}\,2\alpha}{4}}{\text{Sin} - \alpha\,\text{Cos}\,\alpha}\right] \tag{16}$$

$$\rho_4 = \frac{1}{8}\left[\frac{2\alpha - \text{Sin}\,2\alpha}{\text{Sin}\,\alpha - \alpha\,\text{Cos}\,\alpha}\right] \tag{17}$$

where α, which defines location of the neutral axis is given as

$$\alpha = \text{Cos}^{-1}\left[\frac{\sigma - np}{\sigma + np}\right] \tag{18}$$

Equations 12-18 give the required seven equations to solve for seven unknows; namely p, σ, α and ρ_i ($i = 1 - 4$). Fortunately, a simple iteration scheme described below converges rapidly. Iterative solution is started with

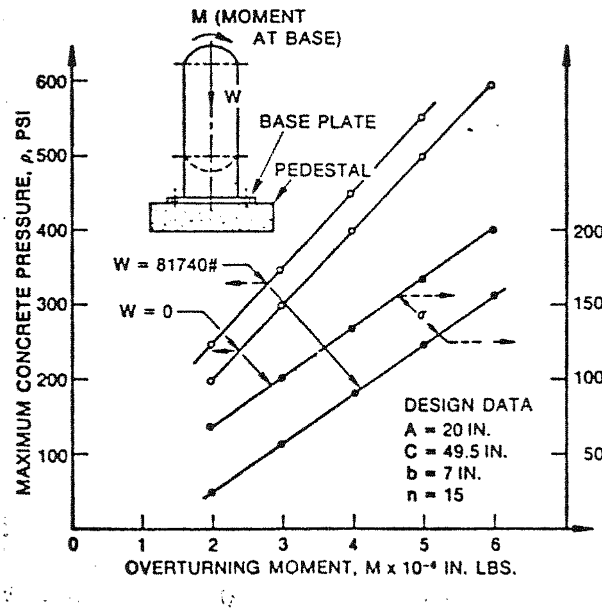

Fig. 3—Maximum concrete pressure and bolt tensile stress from applied loads.

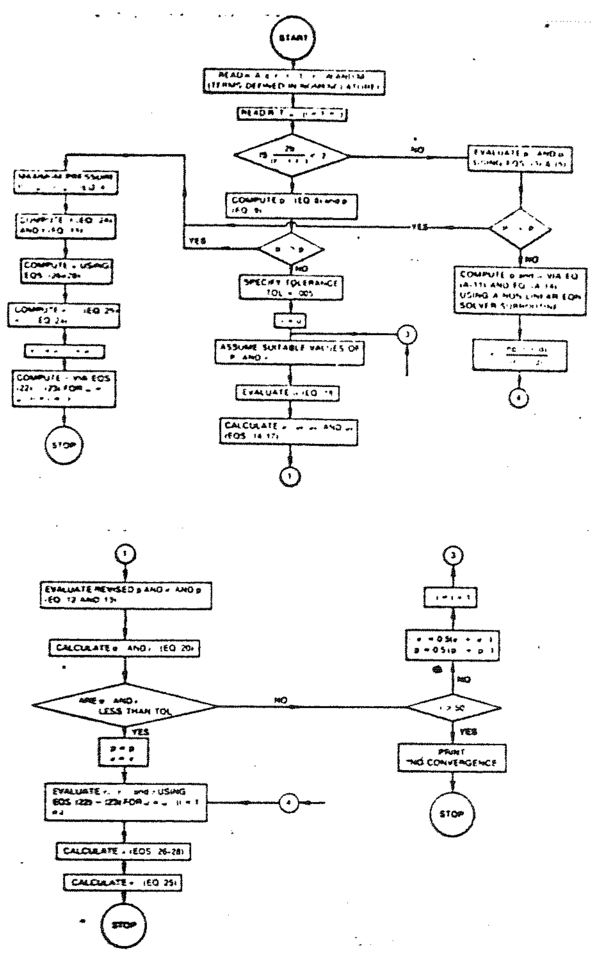

Fig. 4—Flow chart for computer program BEAR.

assumed values of σ and p; say σ_o and p_o. Then α is determined by Equation 18. Knowing α, the dimensionless parameters ρ_1, ρ_2, ρ_3 and ρ_4 are computed. This enables computing corrected values of p and σ (say p_o^1 and σ_o^1) by Equations 12 and 13. Next iteration is started with σ_1 and p_1 where

$$\begin{aligned}\sigma_1 &= 0.5\,(\sigma_o + \sigma_o^1)\\ p_1 &= 0.5\,(p_o + p_o^1)\end{aligned} \tag{19}$$

This process is continued until the errors e_i and ε_i at the ith iteration stage are within specified tolerances. ($e_i = \varepsilon_i = 0.005$ is a practical value), where

$$\begin{aligned} e_i &= \left|\frac{\sigma_i - \sigma_i^1}{\sigma_i}\right| \le 0.005 \\ \varepsilon_i &= \left|\frac{p_i - p_i^1}{p_i}\right| \le 0.005 \end{aligned} \tag{20}$$

Actual numerical tests show that the convergence is uniform and rapid irrespective of the starting values of σ and p.

Axial twist T and lateral shear R. Axial twist and lateral shear induce pure shear in the foundation bolts. Idealizing the foundation bolts by a thin shell of thickness t' as before, such that

$$t' = \frac{A'}{\pi c} \tag{21}$$

where A' = Gross root area of all bolts, the expression for shear stress can be readily developed. The shear stress in x and y directions, τ_x and τ_y for a bolt located at angle ω (Fig. 2) is given by

$$\tau_x = \frac{2T}{cA'} \sin\omega + \frac{R}{A'} \cos\theta$$
$$\tau_y = -\frac{2T}{cA'} \cos\omega + \frac{R}{A'} \sin\theta \qquad (22)$$

Note: T is assumed to be positive when the twist vector is directed vertically downwards.

Hence, combined shear stress τ is given by

$$\tau = (\tau_x^2 + \tau_y^2)^{1/2} \qquad (23)$$

Foundation base plate. The base plate thickness is governed by the radial bending stress, σ_r induced by interfacial concrete pressure and bolt tension. The magnitude of σ_r is calculated with the aid of the following expressions.

Nominal uplift. We have shown that a neutral axis exists; and the pressure distribution is linear along the x-axis, and its maximum magnitude is p defined by Equations 12-18. A practically admissible approximation of σ_r may be obtained by assuming an asymmetric linear pressure distribution about the y-axis with maximum magnitude p instead of the actual loading.

The inside of the plate is assumed to be fixed at the skirt outer circumference. Let the skirt outer radius be r_s, and let

$$\beta = r_s/r_o \qquad (24)$$

This expression is exact within the framework of the aforementioned assumptions and Kirchoff's plate bending theory for the case of no uplift. However, for the case of nominal uplift it is only approximately correct.

It can be shown[4] that the maximum radial stress occurs at the junction of base plate with the skirt and its magnitude is given by

$$\sigma_r = \kappa\, p \frac{r_o^2}{t_b^2} \qquad (25)$$

where t_b is base plate thickness, and κ is defined as

$$\kappa = \frac{1}{32}\left[4(5+\nu)\beta^3 + 2A_1(3+\nu)\beta + \frac{2A_2(1-\nu)}{\beta^3} + \frac{12(1+\nu)}{\beta}\right] \qquad (26)$$

where ν is Poisson's ration, and the geometric constants A_1 and A_2 are defined as

$$A_1 = \frac{-2[4(2+\nu) + (1-\nu)\beta^2(3+\beta^4)]}{(3+\nu) + (1-\nu)\beta^4} \qquad (27)$$

$$A_2 = \frac{-2[4(2+\nu)\beta^4 - (3+\nu)\beta^2(3+\beta^4)]}{(3+\nu) + (1-\nu)\beta^4} \qquad (28)$$

The values of κ for some selected values of β are given in the table below:

β	0.8	0.667	0.5	0.333	0.25	0.2
κ	0.1276	0.3581	0.8679	1.7671	2.6586	3.5107

No uplift. In this case, the pressure distribution is trapezoidal about the y-axis. Thus, the pressure includes two components, namely: the linear component with maximum magnitude p_m, Equation 3, and a uniform pressure p_o, Equation 5.

The radial bending stress σ_r due to the linearly varying component is defined by Equation 25 (with p_m replacing p in Equation 25. The maximum bending stress due to the uniform pressure component p_o is given by[3]

$$\sigma_r = \frac{3p_o r_o^2}{4t_b^2}\left[\frac{-4(1+\nu)\ln\beta - (1+3\nu) + (1-\nu)\beta^4 + 4\nu\beta^2}{(1+\nu) + (1-\nu)\beta^2}\right] \qquad (29)$$

The combined bending stress is found by adding the numerical values of σ_r from Equations 25 and 29.

Example. A skirt-mounted column weighs 40,000 pounds. It has a support base plate 54-in. OD x 40-in. ID x 1.75-in. thick. There are eight bolts spaced on a 49.5 in. bolt circle diameter with a total tensile stress area $n_b A$ = 20 sq. in. The outer diameter of the skirt is 43.25 in. The column is subject to varying levels of wind and piping loads. It is desired to determine maximum concrete pressure p, bolt stress σ and base plate radial stress σ_r for a wide range of overturning moment and vertical thrust values. Specifically, we will perform detailed calculations for M = 5 x 10⁵ lb.-in. or 6 x 10⁶ lb.-in. with an axial thrust W = 81740 lbs. (operating weight).

Case 1: W = 81,750 lbs.; $M = 5 \times 10^5$ lb.-inch. We will first determine if the loading produces no uplift.

Notice, $b = 0.5\,(54 - 40) = 7$ in. and $s = 0.5\,(54 + 40) = 47$ in.

Hence $b/(r_o + r_i) = 7/(27 + 20) = 0.149 < 0.2$; so Relationship 6.a is satisfied.

From Equation 8,

$$p_m = \frac{4M}{\pi s^2 b} = \frac{4\,(5)\,10^5}{\pi\,(47)^2\,(7)} = 41.2 \text{ psi}$$

Similarly, Equation 9 yields

$$p_o = \frac{W}{\pi s b} = \frac{81{,}740}{\pi\,(47)\,(7)} = 79.1 \text{ psi}$$

Since $p_o > p_m$; Equation 10 establishes "no uplift."

Maximum interfacial concrete pressure per Equation 4 is

$$p = p_m + p_o = 41.2 + 79.1 = 120.3 \text{ psi}$$

Our next object is to determine the base plate radial stress. From Equation (24), $\beta = r_s/r_o = 43.25/54 = 0.8$

The maximum base plate stress $\sigma_{r(m)}$ due to the linearly varying component p_m follows from Equation 25. The radial stress in the base plate $\sigma_{r(o)}$ due to the uniform load p_o, is calculated from Equation 29. The combined maximum stress σ_r is found by adding the two values; i.e. $\sigma_r = \sigma_{r(m)} + \sigma_{r(o)}$. The value of κ is calculated using Equations 26, 27 and 28. Poisson's ratio, ν, is generally assumed to be 0.3 for metals. We find that κ = 0.1276.

Since $r_o = 27$ in. and $t_b = 1.75$ in.; from Equation 25,

$$\sigma_{r(m)} = \frac{0.1276\ (41.2)\ 27^2}{1.75^2} = 1{,}251 \text{ psi}$$

Similarly Equation 29 yields, for $p_o = 79.1$ psi,

$$\sigma_{r(o)} = 2{,}545 \text{ psi}$$

Hence the maximum bending stress, σ_r is given by $\sigma_r = \sigma_{r(o)} + \sigma_{r(m)} = 2{,}545 + 1{,}251 = 3{,}796$ psi

Case 2: $W = 81{,}740$ lbs.; $M = 6 \times 10^6$ lb.-inch.

We will first determine if the "no Uplift" condition exists.

From Equation 8: $p_m = \dfrac{4M}{\pi s^2 b} = \dfrac{4\ (6)\ 10^6}{\pi\ (47)^2\ (7)} = 494$ psi

From Equation 9: $p_o = \dfrac{W}{\pi s b} = \dfrac{81{,}740}{\pi\ (47)\ (7)} = 79.1$ psi

Notice that relationship 10 is not satisfied, which rules out "No uplift" hence the solution belongs to the realm of "Normal uplift" for which an iterative solution is obtained using Equations 12-20. The iteration proceeds as follows:

Zeroth iteration $(i = 0)$:

A. Guess initial p and σ:

Let $p_o = 1{,}000$ psi

$\sigma_o = 20{,}000$ psi

$n = 15$ (see Fig. 3)

B. Determine α using Equation 18:

$$\alpha = \cos^{-1} \frac{20{,}000 - 15\ (1{,}000)}{20{,}000 + 15\ (1{,}000)} = 1.4274$$

Note $t = \dfrac{n_b A}{\pi c} = \dfrac{20}{\pi\ (49.5)} = 0.1286$

C. Using Equations 14-17, find $\rho_1 = 1.8336$, $\rho_2 = 2.1606$, $\rho_3 = 0.7849$, $\rho_4 = 4091$.

D. Compute corrected values of p and σ for Zeroth iteration; p_o^1 and σ_o^1, using Equations (12) and (13):

$$p_o^1 = \frac{2\ (81{,}740) + 2.1606\ (0.1286)\ (49.5)\ (20{,}000)}{1.8336\ (49.5)\ (7)}$$

$= 700$ psi

$$\sigma_o^1 = \frac{2\ [(6 \times 10^6) - 81{,}740\ (0.4091)\ (49.5)]}{2.1606\ (0.7849)\ (0.1286)\ (49.5)^2}$$

$= 16{,}260$ psi

Hence, by Equation 20

$$e_o = \left| \frac{20{,}000 - 16{,}260}{20{,}000} \right| = 0.187$$

$$\varepsilon_o = \left| \frac{1{,}000 - 700}{1{,}000} \right| = 0.3$$

Since e_o and ε_o exceed the specified tolerance of 0.005, next iteration (iteration $\#1$) must be started; with initial estimates σ_1 and p_1 evaluated from Equation 19. The rest of the computation is shown in Table 1.

It took nine iterations to converge within the specified tolerance $(e_i, \varepsilon_i \leq 0.005)$. These computations, although manually cumbersome, are inexpensively performed on a digital computer. A flow chart depicting the steps involved in a computer solution is given in Fig. 4. To illustrate the efficiency of computer solution, the maximum bolt stress σ, and concrete pressure p are shown in Fig. 3 as a function of the overturning moment M and vertical thrust W.

Fig. 3 reveals an important result. The pressure p and bolt stress σ are "almost" linear functions of M. This conclusion has been reaffirmed through extensive numerical tests. Thus if one finds p and σ for two (well separated) values of M for a given W, then it is possible to determine p (and σ) approximately for other values of M by linear interpolation. Another important result is the manner of dependence of p and σ on the vertical thrust W. As W is increased p increases, but σ decreases for constant M). Thus, what is "worst" loading for the concrete pressure p is not necessarily the "worst" loading for hold-down bolts. The designer should keep this fact in mind when designing such systems subject to multiple load combinations.

NOMENCLATURE

- A Gross tensile stress area of one foundation bolt, in.^{2}
- A' Gross root area of all bolts, in.^{2}
- b Width of base plate ($b = r_o - r_i$) in.
- c Mean diameter of concrete annulus under base plate, in.
- e_i Error at iteration i (Equation 20)
- d Nominal bolt diameter, in.
- M Overturning moment, in. lbs.
- n Ratio of Young's moduli of bolt material to concrete
- n' Number of threads per inch in foundation bolts
- n_b Number of bolts
- p Maximum concrete pressure, psi.
- p_m Maximum pressure on concrete due to M, psi.
- p_o Uniform pressure due to W, Equation 5, psi.
- R Lateral shear, lbs.
- r Bolt circle radius, $r = 0.5c$, in.
- r_i Inner radius of base plate, in.
- r_o Outer radius of base plate, in.
- r_s Outer radius of skirt, in.
- s Average radius of base plate, $0.5\ (r_o + r_i)$, in.
- T Axial twist, in. lbs.
- t Thickness of fictitious shell based on bolting tensile stress area, in.
- t' Thickness of fictitious shell based on the root area of bolts, in.
- t_b Base plate thickness, in.
- W Vertical thrust, lbs.
- x Variable distance, in.
- α Angle between X-axis and neutral axis, radians
- θ Direction of lateral shear, Figure 2
- β Ratio of outer radius of skirt to outside radius of base plate, r_s/r_o
- τ Combined shear stress in a bolt at angle ω, Fig. 2, psi.
- ρ Dimensionless values, Equations 12-17
- σ Maximum foundation bolt tensile stress, psi.
- σ_r Radial bending stress in base plate, psi
- ε_i Error at iteration i (Equation 20)
- κ Nondimensional ratio, Equation 26
- ω_i Angular orientation of bolt i, radians
- ν Poisson's ratio

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