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# An Approximate Analysis of Foundation Stresses in Horizontal Pressure Vessels

*Saddle supports of horizontally mounted pressure vessels, when subject to seismic and mechanical loads, interact with the foundation in a highly non-linear manner. The maximum foundation concrete pressure, and hold-down bolt stresses are important design considerations which often govern the vessel support geometry. A method is given herein to determine the foundation stresses due to arbitrary imposed loadings. The solution procedure lends itself easily to automated computation—a highly desirable feature—since most nuclear equipment has to be analyzed for a large number of loading conditions.*

## 1 Introduction

In this paper, a method to determine the interfacial concrete pressure, and the foundation bolt stresses for saddle supported pressure vessels is given. The saddle supports of a pressure vessel may be subjected to a variety of loads in its operating life. These loads may be broadly divided into two categories:

1. Time invariant loads; e.g., the operating weight of the equipment, and its attached appurtenances.
2. Time variant loads; e.g., wind loads, inertia loads, forces due to seismic disturbances; reactions transmitted through the nozzles from constrained thermal expansion (contraction) of connecting pipe lines; etc.

The invariant loads are fairly simple to analyze because they are directed vertically downwards and hence produce only a uniform compressive stress on the foundation. Zick [1] gives a comprehensive method, appropriate to a pressure vessel designer, of saddle support design for invariant loads. Variant loads, on the other hand, are more difficult to treat. These loads arise from a host of reasons, and may act coincidentally in numerous combinations. Hence, it is necessary to develop a general formalism for solution which may be computerized to examine the stresses resulting from each conceivable combination. Hence attention is focussed here on determining the stresses in the foundation bolts, and the peak concrete bearing pressure due to an arbitrary set of loadings applied to the pressure vessel. The solution procedure is readily amenable to automation.

A typical pressure vessel supported on two saddles is shown in Fig. 1. One of the two support base plates has slotted holes to accommodate thermal expansion of the shell. We will identify the two supports as "fixed support" and "slotted support."

In Section 2, the support reactions are expressed in terms of the imposed loads. Explicit formulae to correlate the foundation stresses to the support reactions are given in Section 3. Section 4 includes an illustrative example, and a brief discussion of the method is contained in Section 5.

## 2 Determination of Support Reactions

To facilitate analysis, all the imposed loads are resolved along the three coordinate axes as shown in Fig. 1. The origin of the coordinate system, 0, is located on the vessel centerline midway between the supports. The loads are identified as $F_x$, $F_y$ and $F_z$ acting along the $x$, $y$ and $z$ directions. The moments are similarly identified as $M_x$, $M_y$ and $M_z$ with subscripts indicating their axes in accordance with the conventional "right hand screw rule." As stated before, one saddle has slotted holes (elongated in the $x$-direction) to accommodate longitudinal thermal expansion of the vessel. To fix ideas, the slotted support is assumed to be located on the positive $x$-axis and the fixed support is located on the negative $x$-axis. The slotted support is idealized as a frictionless roller in the $x$-direction. The fixed support is

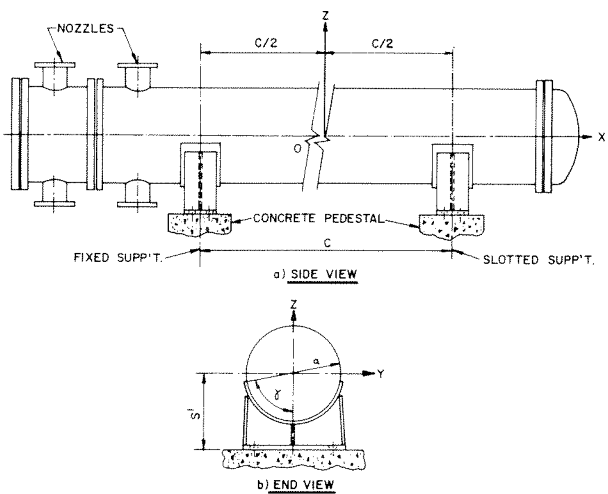


Fig. 1 A saddle supported heat exchanger

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assumed to have its both ends "built-in." This assumption is generally conservative for calculating the foundation stresses. Furthermore, it is realistic to assume that the axial thrust, $F_x$, is resisted by the saddle through a uniformly distributed shear load over the entire circumferential extent of the saddle. Invoking the above assumptions, the foundation reactions and moments can be readily determined by appealing to static equilibrium; as follows:

$$R_{11} = F_x \tag{1}$$

$$R_{12} = 0.5\,F_y - \frac{M_z}{c} \tag{2}$$

$$R_{13} = \frac{F_x(s' - .5s) + M_y}{c} + 0.5\,F_z \tag{3}$$

$$R_{22} = 0.5\,F_y + \frac{M_z}{c} \tag{4}$$

$$R_{23} = -\frac{F_x(s' - .5s) + M_y}{c} + 0.5\,F_z \tag{5}$$

$$M_{12} = \frac{F_x s}{2} + \frac{M_Y}{2} \tag{6}$$

$$M_{11} = M_{21} = -\frac{F_y s'}{2} + \frac{M_x}{2} \tag{7}$$

$$M_{22} = \frac{F_x s}{2} + \frac{M_y}{2} \tag{8}$$

where

$$s = s' - \frac{a \sin\gamma}{\gamma}$$

In the foregoing equations, $R_{ij}$ denotes the reaction and $M_{ij}$ denotes the moment applied on the foundation by the saddle base plate. The first subscript, $i = 1$ (or 2), indicates that the reaction acts at the fixed (or slotted) support; and the second subscript, $j$ (1, 2 or 3), indicates its direction ($x$, $y$ or $z$). $s'$ is the height of the $x$-$y$ plane (Fig. 1); $\gamma$ is the semi-saddle angle, a is the shell outer radius, and $c$ is the distance between support centerlines.

Equations (1–8) give the support reactions as explicit functions of the imposed loads. The next step in the analysis is to develop the relationship between these reactions, and the foundation interfacial concrete pressure and bolt stresses. It suffices to find expressions for concrete pressure due to loadings in one plane, viz., reactions $R_{13}$ and moments $M_{12}$ acting in the $x$-$z$ plane; since effects of loadings in the other vertical plane can be obtained in a similar manner.

## 3 Foundation Stresses

A typical base plate is shown in Fig. 2. The base plate is of rectangular shape with a number of bolt lines. In a general case, let there be $n$ bolt lines parallel to the $y$-axis. The distance of $i$-th bolt line from the right edge of the plate is denoted by $b_i$, $i = 1, n$. Let a moment $M_{12}$ ($= M$) and vertical thrust $R_{13}$ ($= -W$) be applied on the foundation. The concrete base is assumed to be linearly elastic. Noting that the concrete surface is tensionless, and the bolts are effective only in tension, it is necessary to determine the neutral axis. Assuming the base plate to be rigid, the load distribution on the foundation typically looks like Fig. 2 ($b$). Let $d$ denote the distance of the neutral axis from the right edge and $p$ denote the maximum pressure on concrete. Then, if

$$b_{j+1} \geq d > b_j,\; j = 0, n \tag{9}$$

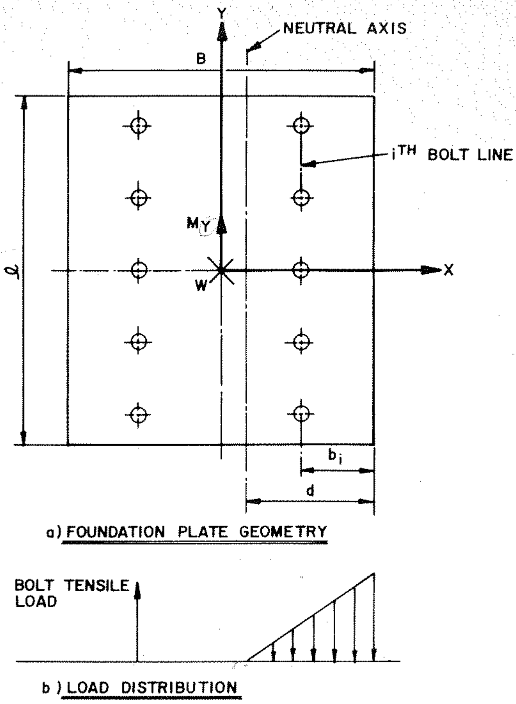


**Fig. 2 Applied loadings on a rectangular base plate**

where $b_0 = 0$, and $b_{n+1} = B$, then vertical force equilibrium requires

$$W = \frac{pd\ell}{2} - \frac{pNA}{d} \sum_{i=j+1}^{n} (b_i - d) \tag{10}$$

Furthermore, moment equilibrium requires

$$M + W(d - 0.5B) = \frac{p\ell d^2}{3} + \frac{pNA}{d} \sum_{i=j+1}^{n} (b_i - d)^2 \tag{11}$$

In the preceding equations, $A$ is the gross tensile stress area available in one bolt line[1] and $N$ is the ratio of the Young's modulus of bolt material to that of concrete. For common structural bolting material—e.g., A325 or A307—and common grades of concrete, $N$ varies from 10 to 15.

The following dimensionless quantities are introduced:

$$\rho_i = \frac{b_i}{B} \tag{12a}$$

$$\rho = \frac{d}{B} \tag{12b}$$

$$\theta = \frac{NA}{\ell B} \tag{12c}$$

$$p^* = \frac{p}{\Lambda} \tag{12d}$$

[1] Tensile stress area, $A'$, of a bolt of nominal diameter $D$, is defined in the *Manual of Steel Construction* [2] as $A' = \pi/4(D - .9743/n')^2$ where $n'$ = number of threads per inch.

## Nomenclature

$A$ = bolt tensile stress area in one bolt line
$a$ = outer radius of shell
$B$ = width of base plate
$b_i$ = distance of $i$-th bolt line (Fig. 2)
$c$ = distance between support centerlines
$d$ = distance of the neutral axis
$F_x, F_y, F_z$ = applied load vectors
$\ell$ = length of saddle base plate
$M$ = applied moment
$M_x, M_y, M_z$ = applied moment vectors
$M_{ij}$ = moment at $i$-th support in the $j$-th direction
$M^*$ = dimensionless moment
$N$ = ratio of Young's moduli
$n$ = number of bolt lines
$p$ = peak concrete pressure
$p^*$ = dimensionless peak concrete pressure
$R_{ij}$ = reaction at $i$-th support in the $j$-th direction
$s'$ = height of centerline
$W$ = vertical load
$W^*$ = dimensionless vertical load
$\gamma$ = semi-saddle angle
$\Lambda$ = nondimensionalized pressure
$\rho$ = dimensionless distance of neutral axis
$\sigma_n$ = maximum bolt tensile stress

$$W^* = \frac{W}{\Lambda \ell B} \qquad (12e)$$

$$M^* = \frac{M}{\Lambda \ell B^2} \qquad (12f)$$

Where $\Lambda$ is an arbitrary nondimensionalizing pressure. Equations (10) and (11) cast in terms of the dimensionless variables become:

$$W^* = \frac{\rho p^*}{2} - \theta p^* \sum_{i=j+1}^{n} \frac{1}{\rho}(\rho_i - \rho) \qquad (13)$$

$$M^* + W^*(\rho - .5) = \frac{\rho^2}{3} p^* + \theta p^* \sum_{j+1}^{n} \frac{1}{\rho}(\rho_i - \rho)^2 \qquad (14)$$

where

$$\rho_j \leq \rho \leq \rho j + 1, j = 0, n \qquad (15)$$

Equations (13) and (14) are combined to yield the following nonlinear equation in terms of $\rho$ only:

$$(0.5\,\rho^2 - \theta\alpha)\,[M^* + W^*(\rho - 0.5)] - W^*\left(\frac{1}{3}\rho^3 + \theta\beta\right) = 0 \qquad (16)$$

where

$$\alpha = \sum_{i=j+1}^{n} (\rho_i - \rho) \qquad (17)$$

$$\beta = \sum_{i=j+1}^{n} (\rho_i - \rho)^2 \qquad (18)$$

Equation (16) is solved for $\rho$ subject to the inequality (15) by using the linear interpolation technique through the method of secants.

The value of $\Lambda$ seems to directly affect the convergence of solutions. As a general rule, keeping $\Lambda$ below the anticipated value of peak concrete pressure ensures convergence.

Having thus determined the location of neutral axis, the peak interfacial concrete pressure $p$ is directly obtained from equation (14). The maximum bolt tensile stress is given by

$$\sigma_n = \frac{1}{d} Np(b_n - d) \qquad (19)$$

It is obvious from physical considerations that solution for equation (16) within the physically meaningful range ($0 \leq \rho \leq 1$) can be found only for a limited range of values of $W$ and $M$. Beyond this range, a neutral axis may not exist. Instead, one of the following two conditions may obtain:

1 **Edging.** For large values of $M$, and relatively small $W$, the entire base plate lifts off the foundation and presses the concrete on one of its edges. This situation will occur when $\rho$ in equation (16) approaches zero. Accordingly, the following relation between $M$ and $W$ will correspond to the onset of edging:

$$M = \frac{W(0.5B\Sigma b_i - \Sigma b_i^2)}{\Sigma b_i} \qquad (20)$$

It should be realized that the aforementioned equation is a necessary, but not a sufficient, condition for edging occurrence.

It can be shown that the maximum stress in the bolts for edging condition is given by

$$\sigma_n = \frac{b_n(M - 0.5WB)}{A\Sigma b_i^2} \qquad (21)$$

The maximum concrete pressure is mathematically infinite.

2 **Complete Compression.** This case exists when the overturning moment is small relative to the vertical load $W$. It can be shown that the pressure distribution is trapezoidal; and its extremal magnitudes are given by

$$p = \frac{W}{B\ell} \pm \frac{3M}{\ell B^2} \qquad (22)$$

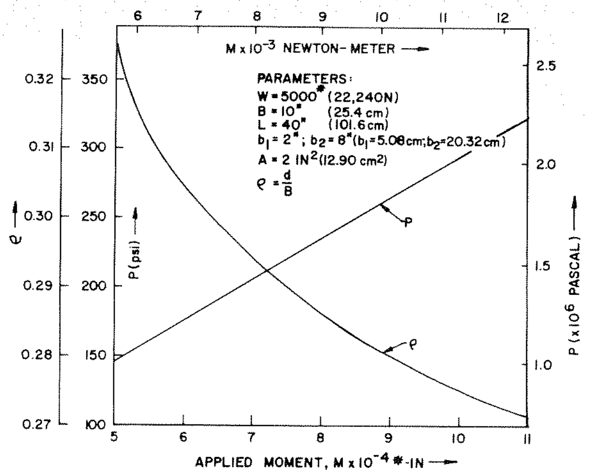


Fig. 3 Variation of $\rho$ and $p$ with applied moment

It should be noticed that from equations (21), in order that tension may exist in all the bolts, the following condition must be satisfied,

$$W^* < 2M^* \qquad (23)$$

Furthermore, in order that some concrete be in compression, it is required that (via equation (22))

$$W^* > 3M^* \qquad (24)$$

It can be shown that equation (24) is a necessary and sufficient condition for complete compression.

## 4 An Example

Consider the case of foundation base plate with an applied vertical load $W$ = 5,000# (22,240N) and a variable overturning moment which ranges from 50,000# in. (5,650N-m) to 110,000# in. (12,430N-m). The geometric parameters are $B$ = 10 in. (0.254m); $\ell$ = 40 in. (1.016m); $n$ = 2; $b_1$ = 2 in. (0.0508m); $b_2$ = 8 in. (0.2032m); $A$ = 2 sq. in. ($1.29 \times 10^{-3}$m²).

The maximum concrete pressure $p$, and neutral axis parameter $\rho$ are calculated, and plotted as functions of the applied moment in Fig. 3.

## 5 Discussion

A method to determine the maximum interfacial concrete pressure, and foundation bolt tension in a rectangular base plate subject to vertical loads and overturning moments is given. The method is particularly useful in designing saddle supported heat exchangers. It is shown that, within a limited range of applied loadings, there exists a neutral axis which can be determined by solving one nonlinear equation in $\rho$ subject to a set of inequality constraints.

## References


1 Zick, L. P., "Stresses in Large Horizontal Cylindrical Pressure Vessels on Two Saddle Supports," *Pressure Vessel and Piping: Design and Analysis*, Vol. 2, ASME, New York, 1972, pp. 959–970.

2 *Manual of Steel Construction*, American Institute of Steel Construction, Seventh Edition, New York 1970, pp. 1265.


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