TUBESHEET ANALYSIS - A PROPOSED ASME DESIGN PROCEDURE

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ABSTRACT

A simplified design procedure is presented to determine tubesheet thickness in U-tube, Floating Head, and Fixed Tubesheet shell and tube exchangers. The procedure is based on classical plate and shell theory and permits a design code oriented manual solution procedure. The configuration analyzed includes an unperforated rim, three different tubesheet head-shell-joint configurations, and arbitrary material properties for each component. Thermal loading due to differential radial expansion and the effect of local pressures acting on the head and shell walls are included.

INTRODUCTION

Analytical procedures for tubesheet analysis have been available for over thirty years. Analytical descriptions are all based on thin plate and shell theories together with simplifying assumptions. For U-tube units, simple solutions, amenable to hand calculations, may be obtained for simply supported or clamped edges. For fixed or floating head units, however, Ber and Bei Bessel functions, reflecting the elastic foundation effect of the tubes, have so far precluded simplified solutions. Evaluation of the tubesheet stress field in exchangers of various styles is presented in (1) which summarizes previously published works in this field (2-10). A tubesheet analysis can always be carried out using the finite element method or special purpose codes for shells of revolution (11); however, presentation of a design code oriented procedure for all classes of tubesheet construction and heat exchanger type remains elusive. The effects of unperforated rim, differential temperature growth in the radial direction, arbitrary combinations of material properties, arbitrary joint configurations and heat exchanger styles so far preclude simple presentation in a code or standard. Typical codes/standards used in various countries for tubesheet design are listed in (12-15); none of these include all of the above considerations. For example, the TEMA design method for fixed tubesheets (12) assumes no unperforated rim, neglects mechanical loading on the shell and headwalls, uses a non-rigorous method for estimating tubesheet edge rotational restraint in a fully integral or one-side integral construction, neglects the effect of radial differential thermal expansion, and currently does not include the effect of variations in the ligament efficiency in the perforated region stress field. Therefore, questionable results may be obtained, especially for tubesheets experiencing high mechanical or thermal load levels. There is a substantial range of configurations for which the TEMA procedure (which has its roots in the analyses by Gardner (10)) yields acceptable design results despite its neglect of the above effects. The TEMA design method is attractive because of its simplicity. For U-tube construction, a single formula suffices to determine the tubesheet thickness, while for fixed and floating head construction, iteration on an assumed tubesheet thickness can be done manually. No calculation of bending stress in the shell or head is provided by TEMA.

The ASME Working Group on Heat Exchanger Equipment has been investing a procedure for determining tubesheet thickness and adjacent structure bending stresses for inclusion in Section VIII, Division I of the ASME Boiler and Pressure Vessel Code (16). This section of the code is generally considered as using a "design by formula" approach. That is, for the specific configurations being analyzed, all necessary information for effecting a solution is presented in formula or chart form.

It requires substantial effort to develop a tubesheet design procedure that includes all construction and loading details heretofore neglected in most codes and standards, yet is simple enough to merit consideration for inclusion in a "design by formula" ASME code. A summary of the development of such a procedure is presented here; the potential user of the proposed method can, therefore, fully understand the basis for final formulas that are being considered for ASME Code Section VIII, Division 1. The method follows the approach of (1) but has been extended to include differential radial thermal growth and cast in a form to present a unified treatment of all heat exchanger styles. In the next section, we outline the important features of the analysis that underlie the proposed code procedure.

TUBESHEET ANALYSIS - INITIAL REMARKS

Figure 1, reproduced from (16), shows the general joint configurations considered for the proposed method outlined here (hereinafter called the proposed ASME method). These details encompass the totality of joint configurations usually found in a U-tube or fixed tubesheet heat exchanger, or at the stationery tubesheet in a floating head heat exchanger. We first consider the unit as a fixed or floating heat exchanger with the tubes acting as stays; subsequently, we specialize the results to U-tube exchangers by neglecting the staying effect of the tubes. The tubesheet perforated region is assumed to have outer radius a; the weakening effect of the perforation array is reflected by using an effective Young's Modulus and Poisson's ratio in this region. Following (1, Sec. 9.3), we can show that the lateral (out-of-plane) deflection of the tubesheet, with respect to its outer edge (r = b in Fig. 1) is given by following differential equation:

$$D^*\nabla^4 w + 2\xi w = Q; \quad D^* = E^*h^3/12(1-\nu^{*2}) \tag{1}$$

In eq. (1), w(r) is the plate deflection in the perforated region, h is the tubesheet thickness, and parameters ξ and Q are given as:

$$\xi = nE_T(d-t)/a^2L; \quad p_S, p_T = \text{shellside, tubeside pressures} \tag{2}$$

$$Q = \frac{p_S A_1 - p_T A_2}{\pi a^2} + \xi\,(\Delta_{T\theta} - \Delta_{S\theta} - \Delta_{TPR} - \Delta_S)$$

E_T, d, t, n are tube Young's Modulus, outer diameter, nominal wall thickness, and number of tubes, respectively. A_1 = tubesheet metal area after drilling and $A_2 = A_1 + n\pi t(d-t)$. L is the tube free length between tubesheets, $\Delta_{T\theta} - \Delta_{S\theta}$ is the net axial thermal growth of the tubes over the shell, and $\Delta_S + \Delta_{TPR}$ represents the change in length of the shell due to axial loads and Poisson ratio effects, respectively. Δ_{TPR} is given as:

$$\Delta_{TPR} = \frac{\nu L}{2E_T t}\quad (p_S\, d - p_T\,[d - 2t]) \tag{3}$$

Δ_S, the shell axial deflection due to mechanical loading, is known for both U-tube and floating head construction but is unknown at this point for fixed tubesheet construction. In deriving eq. (1), we have assumed unit symmetry between tubesheets. While such a restriction is certainly suspect for floating head construction, it is a required assumption if results are to be amenable to hand calculations. Coupling the effects of both tubesheets requires a computer based solution (1). The second term on the left side of Eq.(1) reflects the effect of the tubes acting as stays; to represent a U-tube construction, ξ is set to zero

For U-tube exchangers, ξ=0 and $Q = p_S - p_T$ since the full planform area of the perforated region supports the net load resulting from the pressure difference $p_S - p_T$. The solution to eq. (1) for non-zero ξ involves the Ber and Bei Bessel functions and a parameter x_a (1,10). x_a has the form

$$x_a^{\,4} = 24(1-\nu^2)\,\frac{n\,E_T}{e\,E}\,\frac{(d-t)t}{La}\left(\frac{a}{h}\right)^3; \quad e = \frac{E^*}{E}\,\frac{(1-\nu^2)}{(1-\nu^{*2})} \tag{4}$$

We note that x_a represents the ratio of the axial stiffness of the tube bundle to the bending rigidity of the perforated plate. Therefore, a relatively thick tubesheet and a relatively long bundle of low Young's Modulus tube material produces a low x_a. x_a plays a central role in our further developments. x_a = 0 for U bundles. Prior to presenting detailed design equations, we describe the overall features of the analysis in general terms.

We begin with the solution to the plate equation in the perforated region written in terms of unknown edge moment M_a and edge shear V_a. (See Fig. 2). The plate displacement and rotation at r = a are thus expressible in the symbolic forms $w(a) = w(V_a, M_a)$; $\theta(a) = \theta(V_a, M_a)$. An analysis of the unperforated rim segment (Figs. 2a- 2c) yields a force equilibrium equation relating the axial force in the shell N_S to the edge shear V_a. Thus, $bN_S = aV_a + \ldots$ We can also construct an eqûation for the ring rotation ϕ_R in terms of V_a and M_a. The imposition of joint compatibility relations suffices to determine V_a and M_a. The requirement that $\theta(a) = \phi_R$, at the junction of the perforated and unperforated rim yields M_a in terms of V_a. For U-tube or floating head exchangers, V_a can be directly determined. For a fixed tubesheet unit, a compatibility equation relating tubesheet deflection at the edge of the perforated region and shell axial movement suffices to give V_a in terms of pressure and thermal loads. Once V_a is known, all other quantities of interest are easily determined.

TUBESHEET ANALYSIS - DETAILS OF ANALYSIS

The solution in the perforated region can be expressed in terms of M_a, V_a (see Fig. 2) and can be written symbolically as

$$M_r(x) = M_a\, Q_M(x) + aV_a\, Q_V(x)$$

$$\theta(x)\, D^*/a = M_a\, Z_M(x) + aV_a\, Z_V(x) \tag{5}$$

$$w(x)\, D^*/a^2 = \frac{aQ}{x_a^{\,4}} - M_a\, Z_V(x) - aV_a\, Z_D(x)$$

where $x/x_a = r/a$ and the expressions $Q_M(x)$, $Q_V(x)$, $Z_M(x)$ $Z_V(x)$, and $Z_D(x)$ involve Ber and Bei functions for fixed and floating head heat exchangers. For U-tube exchangers ($x_a \rightarrow 0$), the expressions involve polynomials in (r/a). We note that for U-tube heat exchangers, the edge shear aV_a is a known function of p_S, p_T since overall force equilibrium of the perforated region along the heat exchanger axis yields

$$aV_a = \frac{(p_S - p_T)}{2}\, a^2 = Q\,\frac{a^2}{2} \tag{6}$$

The effective pressure on the tubesheet in the perforated region, including the tube staying effect, if present, can be written in terms of M_a, aV_a as

$$q(x) = \frac{M_a\, x_a^{\,4}}{a^2}\, Z_V(x) + \frac{V_a\, x_a^{\,4}}{a}\, Z_D(x) \tag{7}$$

Figures 2a-c, reproduced from (1), show free body diagrams for the rigid cross-section unperforated rim of the tubesheet with a portion of the adjacent shell and head. It can be shown that for all configurations, the following relation holds between edge shear force V_a and shell axial force N_s:

$$N_s b = aV_a + p_T \frac{b^2}{2} + \Delta p(b^2-a^2)/2; \ \Delta p = p_S - p_T \quad (8)$$

The net moment M_R, acting on the ring at radius R* in the direction of the ring rotation ϕ_R, is now constructed. For example, for two-side integral construction, the net moment has the form: (see Fig. 2a)

$$R^*M_R = - aM_a + aV_a \ (R^* - a) + ,\ldots + a_c(M_c - \frac{h}{2} Q_c) \quad (9a)$$

$$- b \ (M_S - \frac{h}{2} Q_S)$$

For any class of construction, the relation between the ring moment M_R and the ring rotation ϕ_R is

$$\phi_R = 12 \ R^* \ M_R/Eh^3 \ \ell n \ (a_1/a) \quad (9b)$$

Cylindrical shell theory is used to describe the edge displacement and rotation of the shell and head in terms of shell and head edge moments and shears, local pressures p_S, p_T, and free radial thermal expansions (1). Compatibility relations must be satisfied; for example, for two-side integral construction, we must have

$$w_C + \frac{h}{2} \theta_C = \alpha_{TS} \ T_{TS} \ a_c; \ w_S + \frac{h}{2} \theta_S = \alpha_{TS} \ T_{TS} \ b \quad (10)$$

$$\theta_S = - \theta_C = \phi_R \ .$$

α_{TS}, T_{TS} are the coefficient of linear thermal expansion of the tubesheet and the average tubesheet temperature rise above the assembly temperature. Note that we have neglected radial displacement of the tubesheet due to mechanical in-plane loading. Using eq. (10) and the appropriate shell or head edge load solutions, we can determine edge forces and moments, acting on the shell and head, in terms of ring rotation and mechanical and thermal loads. These edge loadings can be eliminated from eq. (9) to yield a relation between ϕ_R, V_a, M_a and the appropriate loadings. The above operations are easily carried out for all joint constructions (see (1)), and the results presented in the general form:

$$\frac{Eh^3}{12} \ (\ln \bar{K} + \mu) \ \phi_R = a^3V_a(K-1) - a \ M_a + Bd_b \ a \ \gamma_B \quad (11)$$

$$+ \ a^3 \ (p_S\gamma_S^{\ **} - p_S^{\ TH} \ \gamma_S^{\ *} + p_T\gamma_T^{\ **} + p_T^{\ TH} \ \gamma_T^{\ *})$$

Appendix 1 defines the terms appearing in eq. (11) in terms of geometry and loading parameters. We note that in deriving the form of eq. (11) used here, we neglect the Poisson effect of axial forces on radial growth of the shell and channel.

A relation between V_a and M_a for general heat exchanger style and joint configuration is obtained by using eq. (11) and (5) in the compatibility equation

$$\theta(a) = \theta(x_a) = \phi_R \quad (12)$$

which yields the result

$$M_a = aV_a \ Q_1 + Q_2 \quad (13)$$

where

$$Q_1 = \frac{K-1 - \Phi \ Z_V(x_a)}{1 + \Phi \ Z_M \ (x_a)}; \ \Phi = \frac{1-\nu^2}{e} \ (\ln \bar{K} + \mu) \quad (14)$$

and

$$Q_2 = \frac{a^2(p_T \ \gamma_T^{\ **} + p_T^{\ TH} \ \gamma_T^{\ *} + p_S \gamma_S^{\ **} - p_S^{\ TH} \ \gamma_S^{\ *}) + Bd_b\gamma_B}{1 + \Phi \ Z_M(x_a)} \quad (15)$$

For a fixed tubesheet exchanger or for the stationary tubesheet of a floating head exchanger, Z_V and Z_M are complex expressions involving Ber and Bei Bessel functions evaluated at x_a. Figures 3a,3b illustrate these functions for $\nu^* = 0.4$. For a U-tube exchanger an appropriate limiting process for $x_a \to 0$ yields the analytical result

$$Z_M(x_a) = 1/1+\nu^*; \ Z_V(x_a) = .25/(1+\nu^*) \quad (16)$$

and we need not resort to any curves to evaluate Z_M, Z_V for U-tube construction.

To completely describe the behavior in the perforated region, we need only determine V_a. For U-tube exchangers or for floating head exchangers, N_S is a known quantity; for many constructions $N_S = p_S b/2$. Therefore, eq. (8) solves for aV_a in the form

$$aV_a = (p_S - p_T) \ a^2/2 \quad (17)$$

For fixed tubesheet construction, N_S cannot be determined from statics alone. An additional requirement on displacement compatibility at the junction between perforated and unperforated region must be applied in the form

$$w(x_a) = (b-a) \ \phi_R = (b-a) \ \theta(x_a) \quad (18)$$

Using eqs. (5), (13), leads to an evaluation for aV_a for fixed tubesheet construction in the form (1):

$$aV_a = \frac{a^2 p_e}{2} = \frac{a \ K \ E_S h_S Q_e - .5\left[J \ \xi \ x_a^{\ 4} \rho_1(x_a)Q_2 - .5\Delta pa^2(K^2-1)\right]}{1 + J \ \xi F_q(x_a)} \quad (19)$$

where J, a parameter reflecting the presence of a shell expansion joint with total spring rate S_j, is given as:

$$J = \frac{1}{1 + 2\pi E_S h_S b/S_j L} \quad (20)$$

The parameters, $\rho_1(x_a)$, $F_q(x_a)$, and Q_e, are given as:

$$\rho_1(x_a) = Z_V(x_a) + (K-1)\, Z_M(x_a)$$

$$F_q(x_a) = QZ1(x_a, Q_1) + (K-1)\, QZ2(x_a, Q_1) \qquad (21)$$

$$Q_e = J\{\Delta_{T\theta} - \Delta_{S\theta} + \lambda_S\, p_S/E_S - \lambda_T\, p_T/E_T\} - .5\, \frac{p_T}{E_S}\frac{b}{h_S}$$

QZ1 and QZ2 are expressions containing Bessel functions; λ_S, λ_T are defined in terms of the unit geometry and material properties

$$\lambda_S = \nu b/h_S + E_S d/E_T t\left(\frac{\nu}{2} + \frac{a^2}{nd^2}\frac{1}{(1-t/d)}\,(1 - nd^2/4a^2)\right) \qquad (22)$$

$$\lambda_T = \frac{\nu d}{2t}\,(1-2t/d) + \frac{a^2}{ndt(1-t/d)}\left(1 - \frac{nd^2}{4a^2}\left(1 - \frac{4t}{d} + \frac{4t^2}{d^2}\right)\right)$$

Examination of eq. (19) for the effective pressure P_e on a fixed tubesheet shows that manual computations are possible, despite the appearance of the Kelvin function expressions, if graphical results can be provided for the expressions $QZ1(x_a, Q_1)$ and $QZ2(x_a, Q_1)$. Figures 4a -4d show typical results for QZ1 and QZ2 (for different values of Q_1) over the practical range of the parameter x_a.

Summarizing the effort to this point, we see that once unit type, geometry, joint configuration, and loading have been specified, an assumption on tubesheet thickness permits computation of x_a, Q_1, Q_2 and the effective pressure $aV_a = a^2P_e/2$. The computations can be carried out without recourse to a computer since we have eliminated any complicating effect of the Kelvin functions by providing graphical results.

Once aV_a is determined for the appropriate heat exchanger style and joint configuration, the ratio M_a/aV_a may be determined from eq. (13) in the form

$$M_a/aV_a = Q_1 + \frac{Q_2}{.5a^2P_e} = Q_3 \qquad (23)$$

It is of interest to note that neglect of Q_2 in eq. (23) is akin to stipulating the neglect of the effect of mechanical loading on the shell, head, and unperforated rim, and the effect of differential radial thermal expansion (between tubesheet, shell, and head) on the stress distribution in the perforated region of the tubesheet. With Q_3 determined, we can evaluate rim rotation $\theta(x_a)$ and effective tube pressure $q(x_a)$; these quantities permit computation of shell and head bending stress and force in the peripheral tubes. It is shown in (1) that $q(x_a)$, $\theta(x_a)$ can be written as

$$q(x_a) = P_e\, QZ1(x_a, Q_3)$$

$$\theta(x_a) = \frac{a^3P_e}{2D^*}\,(Z_V(x_a) + Q_3\, Z_M(x_a)) = \frac{a^3P_e}{D^* x_a^4}\, QZ2(x_a, Q_3) \qquad (24)$$

Thus, graphical data (now using Q_3 as a parameter) enables simple evaluation of edge tube stress in a fixed or floating head unit, and of rotation of the unperforated rim for all styles of heat exchangers. With M_a and aV_a known, the first of eq. (5), can be used to evaluate the maximum value of the radial bending moment in the perforated region of the tubesheet. The maximum bending moment can be written in the form:

$$M_r)_{MAX} = a^2P_e\, F_M(x_a, Q_3)\ ;\ \sigma = \left(\frac{2a}{h}\right)^2\left(\frac{1.5F_MP_e}{\eta}\right) \qquad (25)$$

where

$$2F_M(x_a, Q_3) = \mathrm{Max}\ |(Q_V + Q_3\, Q_M)| \qquad (26)$$

Figures 5a - 5c present typical results for $F_M(x_a, Q_3)$, for $\nu^* = 0.4$, which include all effects of joint configuration and mechanical and thermal loadings. Figures 5a, 5b enable the user to bypass any calculations of Kelvin functions for fixed tubesheet and for floating head construction; for U-tube units, it is easily shown that at $x_a = 0$,

$$2\, F_M = \mathrm{Max}\ \left|\frac{3+\nu^*}{8} + Q_3,\ Q_3\right| \qquad (27)$$

Once tubesheet maximum stress is determined, maximum bending and membrane stress in the head and shell (σ_C, σ_S) (at the connection to the unperforated rim) can be computed. Using eq. (24), and now including the Poisson Ratio effect of axial membrane stresses in the shell and head, yields (for $\nu_C = \nu_S = .3$)

$$\sigma_C = -12\left(1 + \frac{\beta_C h}{2}\right)\frac{E_C}{E}\frac{\beta_C h_C}{2e}\left(\frac{a}{h}\right)^3 P_e\left[Z_V(x_a) + Q_3 Z_M(x_a)\right] + 1.1\,(.85\, p_T + p_T^{TH})\,\beta_C^2\, a_C^2 \qquad (28)$$

$$\sigma_S = 12\left(1 + \frac{\beta_S h}{2}\right)\frac{E_S}{E}\frac{\beta_S h_S}{2e}\left(\frac{a}{h}\right)^3 P_e\left[Z_V(x_a) + Q_3 Z_M(x_a)\right] + 1.1\,\left(p_S - .3\,\sigma_{LS}\frac{h_S}{b} + p_S^{TH}\right)\beta_S^2 b^2 \qquad (29)$$

the axial membrane stress in the shell

$$\sigma_{LS} = \frac{a^2p_e}{2bh_S} + \frac{p_Tb}{2h_S} + \frac{.5\Delta pa}{Kh_S}\,(K^2 - 1);\ \sigma_{LC} = \frac{P_Ta_c}{2h_c} \qquad (30)$$

and the axial stress in the channel is

$$\beta_C^{\ 4} = 3(1-\nu_C^2)/a_C^2h_C^2;\ \beta_S^{\ 4} = 3(1-\nu_S^2)/b^2h_S^2 \qquad (31)$$

It is important to note that in the computation of σ_C, σ_S, the Poisson ratio effects must be retained; we have, however, neglected these effects in the computation of M_a, V_a. Numerical comparisons with finite element solutions bear out the validity of our neglecting or retaining these terms at certain points in the development.

REMARKS

We have outlined a complete solution procedure to evaluate a proposed tubesheet, shell, channel configuration that can be carried out manually despite the appearance of Kelvin functions in the solution. Appropriate graphical data is easily provided in a suitable form for inclusion in code procedures. All styles of heat exchangers, with arbitrary joint configurations, and with arbitrary geometric and material properties can be treated with no increase in computational complexities. The effects of radial differential thermal expansion in all heat exchanger styles is included, as well as the effects of local pressure on the shell, channel, and unperforated rim.

The procedure has been compared with numerical solutions using a special purpose shell analysis code (17). Good agreement with the numerical results has been obtained in all areas of the structure although the neglect of membrane stresses in the perforated region may lead to a slightly non-conservative prediction of stress in the perforated region. Based on the above presentation, a step by step design procedure is being prepared for presentation to the appropriate ASME code committees.

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APPENDIX I - DEFINING RELATIONS FOR TERMS IN EQ. (11)

$\bar{K} = a_1/a$; $K = b/a$; $K_C = a_C/a$

$D_S = E_S h_S^3/12(1-\nu_S^2)$; $D_C = E_C h_C^3/12(1-\nu_C^2)$

$\beta_S^4 = 3(1-\nu_S^2)/b^2 h_S^2$; $\beta_C^4 = 3(1-\nu_C^2)/a_C^2 h_C^2$

ε_S = 1 shell side integral; ε_C = 1 channel integral

= 0 shell side gasketed; = 0 channel gasketed

$$\frac{Eh^3\mu}{24} = \varepsilon_S\,\{\beta_S b D_S[1+\beta_S h+\beta_S^2 h^2/2]\} + \varepsilon_C\,\{\beta_C a_C D_C\,[1+\beta_C h+\beta_C^2 h^2/2]\}$$

$\gamma_S^* = 2\beta_S^2 D_S(1 + \beta_S h)K^3/E_S h_S$; $\gamma_T^* = 2\beta_C^2 D_C(1+\beta_C h)K_C^3/E_C h_C$

$$\gamma_S^{**} = (K^2-1)(K-1)/4 - \varepsilon_S \gamma_S^*$$

$$\gamma_T^{**} = (K_C^2-1)(K_C+1)/4 - (K_C^3-K)/2+\varepsilon_C \gamma_T^*$$

γ_b = 0 (both sides integral)

= $K_C - d_b/a$ (shell side integral - tube side gasketed)

= $K - d_b/a$ (tube side integral - shell side gasketed)

= $K_C - K$ (both sides gasketed)

$$p_T^{TH} = E_C h_C/a_C\,[\alpha_C T_C - \alpha_{TS} T_{TS}]$$

$$p_S^{TH} = E_S h_S/b\,[\alpha_S T_S - \alpha_{TS} T_{TS}]$$

B = bolt load per unit of circumferential length around a circle of radius d_b