APPLICATION OF TRANSIENT ANALYSIS METHODOLOGY TO HEAT EXCHANGER PERFORMANCE MONITORING

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ABSTRACT

A transient testing technique is developed to evaluate the thermal performance of industrial scale heat exchangers. A Galerkin-based numerical method with a choice of spectral basis elements to account for spatial temperature variations in heat exchangers is developed to solve the transient heat exchanger model equations.

Testing a heat exchanger in the transient state may be the only viable alternative where conventional steady state testing procedures are impossible or infeasible. For example, this methodology is particularly suited to the determination of fouling levels in component cooling water system heat exchangers in nuclear power plants. The heat load on these so-called "component coolers" under steady state conditions is too small to permit meaningful testing. An adequate heat load develops immediately after a reactor shutdown when the exchanger inlet temperatures are highly time-dependent. The application of the analysis methodology is illustrated herein with reference to an in-situ transient testing carried out at a nuclear power plant. The method, however, is applicable to any transient testing application.

NOMENCLATURE

A	Heat Exchanger Area/Unit Length
A_n, B_n	Time Dependent Fourier Coefficients (Eqns. 15,16)
K_s, K_t	Shell, Tube Functions Defined by Equation 14
L	Heat Exchanger Length
m_s, m_t	Shell, Tube Mass Flow Rate x Stream Heat Capacity
p_s, p_t	Shell, Tube Heat Exchanger Thermal Holdup/Unit Length
T, t	Shell, Tube Temperatures as Functions of Time and Position in the Heat Exchanger
U	Overall Heat Transfer Coefficient
x	Heat Exchanger Length Co-ordinate
θ,ϕ	Shell, Tube Temperature Deviations
τ	Time
{ }	Column Vector
[ ]	Square Matrix

Subscripts

s,t	Shell, Tube
ss	Steady State
0	At Heat Exchanger Position x = 0
L	At Heat Exchanger Position x = L

INTRODUCTION

Interest in performance testing of heat exchangers has increased dramatically over the last few years. It is well known that heat transfer surfaces are prone to deposition of various foreign materials, e.g., silt, biological growth, corrosion products, and scale. These deposits gradually build up, adding heat transfer resistance to the exchanger surfaces, a phenomenon commonly referred to as fouling. Heat exchangers are designed for expected levels of fouling depending on their planned application (TEMA Standard, 1988). The net result of design fouling level is an increase in heat transfer area to compensate for the reduced heat transfer coefficient during the design phase. Generally, a compromise must be made to minimize both expected maintenance costs and the original heat exchanger size. Since heat exchangers are not generally sized to allow continuous operation without cleaning, periodic maintenance becomes a necessity. Monitoring the performance of industrial heat exchangers can be an important part of a preventive maintenance program. Measuring actual performance minimizes cleaning frequency and eliminates unnecessary maintenance. Test results can be trended to predict when component performance will become limiting and to allow timely scheduling of cleaning or other maintenance activities. At the same time, testing ensures that heat exchangers which perform a critical function related to plant safety, operability, or production are available to meet their design requirements at all times.

This important monitoring function is implemented by conducting periodic in-situ tests of heat exchanger performance during controlled plant conditions. Theoretically, the simplest test to perform and evaluate is one conducted at steady state conditions. Establishing steady state conditions reduces the data gathering requirements and simplifies the analysis. More complex considerations, such as thermal inertia, are eliminated from the evaluation. A recent EPRI Guideline (Stambaugh et al.) outlines a basic test and analysis methodology and provides test criteria to ensure meaningful results. Statistical techniques to analyze steady state test data may be found in recent publications (Singh et al., 1994). However, in practice, steady state testing often is not feasible or practical. Many heat exchangers normally operate at heat loads which represent only a fraction of their rated capability. The normal heat load and the resulting differential temperatures may be inadequate to provide statistically meaningful analysis of the results. Conversely, it may be physically possible to establish steady state conditions, but downstream operations will be negatively impacted. In such circumstances, an alternative approach to exchanger testing developed in this paper can be utilized.

An important problem which motivated the present work on transient heat exchanger testing is the development of a procedure to determine the fouling levels in component coolers in nuclear power plants. Immediately after a reactor shutdown, the component coolers are lined up to cooldown the hot reactor. During the short term transient reactor cooling period, the component cooler operates with a relatively large heat duty and high temperature differentials before it reaches a more or less steady operation at reduced heat duties to remove the long-term reactor fuel decay heat. Noting that the reliability of heat exchanger performance test results are much greater under conditions of high heat duties, there is a significant incentive to collect and analyze operating data during the transient reactor cooling time period.

Heat exchanger testing under transient conditions has been reported in recent literature. In 1986, Mullisen and Loehrke developed a curve matching methodology to estimate the average heat transfer coefficient from transient response of regenerators. In another related application, Courville and Beck (1989) solved the problem of estimating thermal conductivities of roof insulations from in-situ transient testing in combination with least squares minimization techniques. An important requirement for analyzing transient test data is to have developed and solved the appropriate transient heat exchanger model equations. In some cases analytical solutions are available (for example, solution to the transient equations of direct transfer crossflow heat exchangers reported in 1988 (Spiga and Spiga)). Many of the developed methodologies assume specific inlet temperature functions or require complex, computer based solution. Duplication of a specific temperature profile, such as a step change, at the inlet of a large industrial heat exchanger may be impractical. Therefore, this work first focuses on evaluation of an arbitrary inlet temperature or flow defined as a function of time.

A practical problem motivated the development of a mathematical model to predict the transient performance of liquid/liquid, countercurrent flow heat exchangers. As discussed earlier, power plant component coolers experience large heat loads during periodic plant cooldowns. In this scenario, the process side temperature decreases substantially over time. If a heat exchanger is operated under constant flow conditions, but with time varying inlet temperatures, then the transient response depends essentially on the unknown overall heat transfer coefficient and known heat exchanger parameters, e.g., heat transfer area, tube/shell dimensions, materials, etc. A mathematical model to determine the resulting transient outlet temperatures of a counterflow heat exchanger was developed.

Inasmuch as the transient heat exchanger mathematical model was central to the development and implementation of the transient exchanger testing methodology, the next two sections provide a detailed description of the model and the numerical techniques used to solve the transient heat balance equations. The next section after numerical solution details an actual transient test which was conducted to implement the methodology. This is followed by a section which describes development of the transient testing methodology and the test results. The last section summarizes the important conclusions of this work.

TRANSIENT RESPONSE OF COUNTERCURRENT HEAT EXCHANGERS

General analytical solutions to countercurrent transient processes have been known for quite some time (Jaswon and Smith, 1954). Unfortunately, the form of the solution is quite complex, requiring unwieldy computer calculations as pointed out by Tan and Spinner (1984), who instead reported a numerical solution procedure using the method of characteristics. Nevertheless, a number of complex analytical solutions under various restrictive conditions have been reported in the literature recently (Li (1986), Spiga and Spiga (1988), Gvozdenac (1987)). In the present work, a numerical approach is adopted to solve the transient heat balance equations valid for the case of liquid/liquid counterflow units. Other than this, no other restriction is imposed on the numerical procedure with regard to the form of disturbances to either inlet temperatures, flows or overall heat transfer coefficient. These quantities are all assumed to be specified functions of time.

Figure 1 shows a differential length of a countercurrent heat exchanger with both fluid streams having their appropriate positive direction. An energy balance applied to each stream yields the governing equations given below.

$$p_s \frac{\partial T}{\partial \tau} = - m_s \frac{\partial T}{\partial x} - UA(T-t) \qquad (1)$$

$$p_t \frac{\partial t}{\partial \tau} = m_t \frac{\partial t}{\partial x} + UA(T-t) \qquad (2)$$

In the equations above, T and t are the respective shell and tubeside temperatures, p_s and p_t are the shell and tube thermal capacities per unit length of the heat exchanger, m_s and m_t are the shell and tube heat flow parameters (mass flow rate * stream heat capacity), U is the overall heat transfer coefficient, A is the heat exchanger area per unit length of the exchanger, x and τ are the length and time co-ordinates. The model equations are identical to those reported by Li (1986) for co-current heat exchangers.

In the above equations, the thermal capacity of the heat exchange metal has been neglected. This assumption is valid for liquid-liquid type heat exchange processes where the liquid thermal capacity is generally much higher than the thermal capacity of the heat exchanger mass (Li, 1986). Also, all thermo-physical properties are assumed constant. The steady state temperature fields for the countercurrent heat exchanger are obtained from the following equations:

$$O = -\bar{m}_s \frac{\partial \bar{T}_{ss}}{\partial x} - \bar{U}A\,(\bar{T}_{ss} - \bar{t}_{ss}) \qquad (3)$$

$$O = \bar{m}_t \frac{\partial \bar{t}_{ss}}{\partial x} + \bar{u}A\,(\bar{T}_{ss} - \bar{t}_{ss}) \qquad (4)$$

where, $\bar{T}_{ss}$ and $\bar{t}_{ss}$ are the steady state shell and tube temperature distributions.

Note that we permit m_s, m_t, U to be functions of time since the flow rates can vary during the transient. We define $\theta(x,\tau)$, $\phi(x,\tau)$ as the deviation, from the steady state distribution, of the shell and tubeside streams, respectively. Then

$$T(x,\tau) = \bar{T}_{ss}(x) + \theta(x,\tau) \qquad (5)$$

$$t(x,\tau) = \bar{t}_{ss}(x) + \phi(x,\tau) \qquad (6)$$

At $t > 0$, we assume the individual inlet shell and tube temperature transients to be specified by

$$\theta(0,\tau) = \theta_o(\tau) \qquad (7)$$

$$\theta(L,\tau) = \phi_L(\tau) \qquad (8)$$

By virtue of the definition of θ, ϕ as changes from steady state, it is clear that

$$\theta(x,0) = \phi(x,0) = 0 = \theta_0(0) = \phi_L(0) \qquad (9)$$

Using Equations (5), (6) in Equations (1), (2), and introducing the steady state Equations (3), (4) yields

$$p_s \frac{\partial \theta}{\partial \tau} = - m_s \left(\frac{\partial \bar{T}_{ss}}{\partial x} + \frac{\partial \theta}{\theta x}\right) - UA(\theta - \phi) - UA(\bar{T}_{ss} - \bar{t}_{ss}) \qquad (10)$$

$$p_t \frac{\partial \phi}{\partial \tau} = m_t \left(\frac{\partial \bar{t}_{ss}}{\partial x} + \frac{\partial \phi}{\partial x}\right) + UA(\theta - \phi) + UA(\bar{T}_{ss} - \bar{t}_{ss}) \qquad (11)$$

We next introduce the following parameters:

$$\alpha_{11}^o = -\frac{m_s}{p_s} \qquad \alpha_{12}^o = -\frac{UA}{p_s}$$

$$\alpha_{21}^o = \frac{m_t}{p_t} \qquad \alpha_{22}^o = \frac{UA}{p_t}$$

Equations (10) and (11) can now be written as:

$$\frac{\partial \theta}{\partial \tau} = \alpha_{11} \frac{\partial \theta}{\partial x} + \alpha_{12}(\theta - \phi) + K_s(x) \qquad (12)$$

$$\frac{\partial \phi}{\partial \tau} = \alpha_{21} \frac{\partial \phi}{\partial x} + \alpha_{22}(\theta - \phi) + K_t(x) \qquad (13)$$

here,

$$K_s(x) = \alpha_{11}\,\bar{T}'_{ss} + \alpha_{12}(\bar{T}_{ss} - \bar{t}_{ss})$$
$$K_T(x) = \alpha_{21}\,t'_{ss} + \alpha_{22}(\bar{T}_{ss} - \bar{t}_{ss}) \qquad (14)$$

The numerical procedure to solve the foregoing partial differential equations is presented next.

SOLUTION OF THE MODEL EQUATIONS

Recently, Romie (1984) has reported the numerical solution of transient countercurrent heat exchanger model equations using finite difference techniques. In the present work, a Galerkin based method (Mikhlin, 1964) with a choice of spectral basis functions is developed to solve the model equations. This procedure was selected because of its important advantage over finite difference methods in terms of its exponential rate of convergence (Gottlieb and Orszag, 1986). In other words, for a fixed level of specified error in the numerical approximation, a fewer number of spectral basis functions will be required as compared to the number of finite difference nodes to solve the partial differential equations. Thus, the numerical effort expended in terms of computational time and computer memory resources is expected to be much smaller.

The Galerkin Method is applied to change the partial differential equations to a set or ordinary differential equations in time. Recalling that

$$\theta\ (0,\tau) = \theta_o\ (\tau)\ ;\ \phi\ (L,\tau) = \phi_L\ (\tau)$$

we note that all boundary conditions of the problem are satisfied by assuming the temperature changes $\theta(x, \tau)$, $\phi(x, \tau)$ to be numerically approximated by:

$$\hat{\theta}\ (x,\tau) = \theta_o\ (\tau)\ (1 - \frac{x}{L}) + \sum_n A_n\ (\tau) \sin \frac{n\pi x}{2L} \tag{15}$$

$$\hat{\phi}\ (x,\tau) = \phi_L \frac{x}{L} + \sum_n B_n\ (\tau) \sin \frac{n\pi}{2} (1 - \frac{x}{L}) \tag{16}$$

where, $\hat{\theta}$ and $\hat{\phi}$ are the respective approximate functions of θ and ϕ and A_n, B_n are unknown constants. We now use Galerkin's Method to set forth a set of ordinary differential equations (in time) to solve for $A_n(\tau)$, $B_n(\tau)$. Once these functions are determined, we can evaluate $\hat{\theta}$, $\hat{\phi}$ at any location x for all values of time. Note that the use of sin functions (a complete set) ensures convergence to the exact solution as the number of terms in the series is increased.

In the first step of the Galerkin procedure, the approximation functions for θ and ϕ (Equations (15) and (16)) are substituted in equations (12) and (13). Note that for *no* choice of A_n and B_n, Equations (12) and (13) can be satisfied identically. Instead, the Galerkin procedure provides a method to determine the unknown coefficients which minimize the total error in the solution. to this end, residual functions $R_\theta\ (x,\tau)$ and $R_\phi\ (x,\tau)$ obtained from Equations (12) and (13) are defined below:

$$R_\theta\ (x,\tau) = p_s \frac{\partial\hat{\theta}}{\partial\tau} + m_s \left(\frac{\partial \bar{T}_{ss}}{\partial x} + \frac{\partial\hat{\theta}}{\partial x}\right) + UA\ (\hat{\theta} - \hat{\phi}) + UA\ (\bar{T}_{ss} - \bar{t}_{ss}) \tag{17}$$

$$R_\phi(x,\tau) = p_t \frac{\partial\hat{\phi}}{\partial\tau} - m_t \left(\frac{\partial \bar{t}_{ss}}{\partial x} + \frac{\partial\hat{\phi}}{\partial x}\right) - UA\ (\hat{\theta} - \hat{\phi}) + UA\ (\bar{T}_{ss} - \bar{t}_{ss}) \tag{18}$$

The Galerkin criteria requires the residual function be orthogonal to *each* of the chosen spectral basis functions in Equations (15) and (16). This orthogonality condition is expressed as:

$$\int_0^L R_\theta\ (x, \tau) \sin \frac{n\pi x}{2}\ dx = 0 \tag{19}$$

$$\int_0^L R_\phi\ (x, \tau) \sin \frac{n\pi}{2} (1 - \frac{x}{L})\ dx = 0 \tag{20}$$

Equations (19) and (20) yield the following set of ordinary differential equations expressed in Einstein's notation (Summation implied on a repeated index):

$$\alpha_{ij} \frac{dA_j}{d\tau} + \beta_{ij} A_j + \gamma_{ij} B_j = F_i \tag{21}$$

$$\delta_{ij} \frac{dB_j}{d\tau} + \epsilon_{ij} A_j + \mu_{ij} B_j = G_i \tag{22}$$

The coefficients α_{ij}, β_{ij}, γ_{ij}, F_i, δ_{ij}, ϵ_{ij}, μ_{ij} and G_i are all readily determined from the Galerkin's orthogonality conditions (Equations (19) and (20) as summarized in the Appendix herein).

Equations (21) and (22) form 2N first order differential equations in time for N variables A_n, B_n. By virtue of Equation (9), the initial conditions are $A_n(o) = B_n(o) = 0$ for n = 1,2,..N. A program has been written to solve the equations by stepping along in time using a Runge-Kutta solution. This completes the analysis of the transient countercurrent heat exchange process and the necessary relationships required to obtain and solve the model equations. In the marching solution, it is assumed that $\theta_o(\tau)$ and $\phi_L(\tau)$ are specified functions of time τ.

DESCRIPTION OF THE TRANSIENT HEAT EXCHANGER TESTING AT CALVERT CLIFFS NUCLEAR POWER PLANT

The component cooling water system (CCW) at Calvert Cliffs Nuclear Power Plant essentially consists of a coupled system of one shutdown cooler (SDCHX) and one component cooling water heat exchanger (CCWHX) used to cooldown the reactor vessel after a plant shutdown. A schematic outline of the system tested is shown in Figure 2. The SDCHX exchanger is a two-tube pass heat exchanger with the hot reactor water flowing on the tubeside and CCW fluid on the shellside. The heat intake by the CCW fluid from the hot reactor fluid in the SDCHX exchanger is finally rejected to the salt water (SW) stream in the CCWHX exchanger. The CCWHX exchanger is a countercurrent shell and tube exchanger with SW flow in the tubeside and CCW flow in the shellside.

During a recent shutdown at Calvert Cliffs plant in February, 1993, the component cooling water system was lined up for cooling the reactor vessel when initially the reactor water to SDCHX inlet was at 245°F. Subsequently, for about six hours, an automatic data acquisition system recorded all inlet/outlet temperatures and flows of both the CCWHX and SDCHX exchangers. As a result of the reactor cooldown, the temperature of the reactor water to SDCHX exchanger continuously decreased during the six hour period from 245°F to 145°F (i.e., a drop of 100°F). This large transient temperature variation induced a significant variation in the outgoing CCW stream temperature from the SDCHX exchanger. As a result, the CCWHX exchanger operation during this six hour period was also under unsteady conditions. In this work, the simpler countercurrent CCWHX exchanger has been selected to develop and demonstrate the transient testing procedure. In principle, the procedure is valid for transient testing of the 2-tube pass SDCHX exchanger and in fact any type of multi-pass heat exchanger. The difference arises in the additional complexity involved in developing a numerical procedure to predict the transient response of multi-pass heat exchangers.

The main effect of unsteady heat exchanger operation can be understood by including the thermal mass of the CCWHX heat exchanger into the heat transfer model. The hot fluid, as usual, gives up heat to the cold fluid as it flows through the heat exchanger. A temperature transient implies that the heat exchange metal and the tube/shell fluid temperatures are time varying. Thus, the combined thermal mass of the heat exchanger metal, the tubeside fluid and the shellside fluid absorb a fraction of the heat exchanged between the two streams if, for example, the temperatures are increasing with time.

An examination of the unsteady CCWHX exchanger test data collected during the six hour period reveals that the heat exchanger inlet/outlet temperatures were smoothly varying with time for a major duration of the test if the first and last hour of the testing period (i.e., startup and end of test) are not included. The CCWHX shell and tube flows are also reasonably steady during this test period. The unsteady CCWHX test data in an approximately three hour time slot is chosen for subsequent analysis. The transient test data is sampled at 11 instants in time in this chosen three hour time slot at approximately uniformly spaced intervals. The CCWHX shell and tube inlet and outlet temperatures and flow rates are recorded at the 11 time instants.

In Figure 3, the four inlet/outlet temperatures are shown as a function of time. From the recorded data, the CCWHX individual shell and tubeside heat duties are calculated and plotted as a function of time as shown in Figure 4. The recorded transient data is analyzed, using the numerical procedure developed earlier to predict exchanger performance, to determine the overall heat transfer coefficient of the CCWHX exchanger. The determination of the overall heat transfer coefficient from the numerical solution is described below.

OUTLINE OF THE ANALYSIS PROCEDURE

If the shell and tubeside inlet temperatures to a heat exchanger are unsteady, then the transient heat exchanger model will predict the time varying outlet temperatures. The predicted outlet temperatures are functions of the *unknown* overall heat transfer coefficient. An important assumption required to undertake this analysis is the notion of this single overall coefficient U_o which can effectively fit the experimentally measured transient data. Inherently, this assumption implies that variations in U_o due to:

(a) fluid property variations due to temperature changes
(b) tubeside flow variations, and
(c) shellside flow variations

are small and can be neglected.

Thus, from the recorded inlet temperatures and flows as functions of time, the outlet CCWHX temperatures are calculated using the numerical procedure developed to solve the transient model equations starting from an initial guess of the unknown parameter U_o. The relevant CCWHX exchanger thermal/hydraulic design data required by the computer program is summarized in Table 1. The computed outlet tube and shellside temperatures are compared with the experimentally measured CCWHX outlet temperatures using the following sum of squares error criteria:

$$SSQ(U_o) = \sum_{K=1}^{11} [T_{cal,K} - T_{exp,K}]^2 \qquad (23)$$

where, $T_{cal,K}$ and $T_{exp,K}$ are the respective calculated and experimental values of the outlet shell or tubeside temperatures at different time instants and SSQ is the sum of squares error function.

Note the error in the fit is a function of the unknown parameter U_o. By varying this parameter, an optimum or best estimate of U_o which minimizes the *total* error is obtained. This optimization process is shown in Figure 5. The individual sum of squares error for the shell and tube outlet temperatures as well at the total (tube plus shell) error is plotted as a function of the unknown parameter U_o. It is readily apparent that the sum of squares criteria is minimized for each of the individual and total errors within a narrow range of the parameter U_o. The improved matching of the transient experimental data near the optimum value of the parameter U_o is illustrated in Figure 6 where the experimental component cooling water outlet temperature as a function of time is compared with predicted outlet temperature profiles for different values of U_o. Note how the calculated temperature curve moves closer and closer to the experimental profile as the optimum is approached.

The best estimate of the overall heat transfer coefficient is given by the value of U_o which minimizes the *total* error. This criteria takes into account the complete set of available data (inlet/outlet shell and tubeside temperatures and flows) without giving any extra weightage or bias to the experimental data values. The best estimate U_o for the CCWHX exchanger is calculated to be 319.1 Btu/(ft^2)(hr)(°F) under test conditions. From this actual coefficient and a calculated clean coefficient for the CCWHX exchangers of 466.4 Btu/(ft^2)(hr)(°F) a fouling level of 0.00099 ft^2 hr °F/Btu has been determined by this procedure (see Table 2).

CONCLUSION

A general transient heat exchanger performance testing and evaluation procedure applicable for any single or multipass heat exchange equipment has been developed. This procedure has been illustrated by its implementation on the CCWHX exchanger at the Calvert Cliffs Nuclear Power Plant to determine its fouling level. The transient testing procedure is applicable in those instances when conventional steady state based testing methodologies are not applicable. Heat exchangers in component cooling water systems at nuclear power plants are industrially important candidate cases for application of this methodology. This procedure is also expected to be useful for performance testing of heat exchangers in the chemical, refining and petrochemical and other industry sectors which employ heat exchangers subject to time-dependent fouling.

Table 1: CCWHX Thermal-Hydraulic Data

Heat Transfer Area	=	5720.4 ft^2
Exchanger Length	=	29.7 ft
Tube ID/OD	=	0.652"/0.75"
Shell Thermal Holdup	=	226.9 Btu/(°F)(ft)
Shell Flow Rate	=	2,374,000 lb/hr
Tube Thermal Holdup	=	176.8 Btu/(°F)(ft)
Tube Flow Rate	=	2,216,000 lb/hr

Table 2: Estimation of CCWHX Fouling

From Heat Transfer Correlations

Tube Inside Film Coefficient (h_{ii}) = 915.6 Btu/(ft^2) (hr)(°F)

Shell Outside Film Coefficient (h_{oo}) = 1306.9 Btu/(ft^2)(hr)(°F)

Tube Metal Wall Coefficient (h_{to}) = 8119.9 Btu/(ft^2)(hr)(°F)

Fouling Calculations

Best Estimate Overall U_o = 319.1 Btu/(ft^2)(hr)(°F)

Fouling Resistance ($1/h_{fo}$) =

$$[\frac{1}{U_o} - \frac{1}{h_{oo}} - \frac{1}{h_{to}} - \frac{1}{h_{ii}}(ID/OD)]$$
$$= 0.00099\ ft^2\ hr\ {}^\circ f\ /\ Btu$$

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APPENDIX

Steady State Temperature Profiles

The initial steady state shell and tube axial temperature profiles in the countercurrent heat exchanger are given by the following equations:

$$\bar{T}_{ss}(x) = \bar{T}_{os} - \frac{\bar{T}_{os} - t_{LT}}{1 - \beta e^{-\lambda L}}\left[1 - e^{-\lambda x}\right] \qquad (A.1)$$

$$\bar{t}_{ss}(x) = \bar{T}_{os} - \frac{\bar{T}_{os} - t_{LT}}{1 - \beta e^{-\lambda L}}\left[1 - \beta e^{-\lambda x}\right] \qquad (A.2)$$

where

$$\begin{aligned} \bar{T}_{os} &= \bar{T}_{ss} \text{ at } x = 0 \\ \bar{t}_{LT} &= \bar{t}_{ss} \text{ at } x = L \\ \beta &= \frac{\bar{m}_s}{\bar{m}_t} \\ \text{and, } \lambda &= \frac{UA}{\bar{m}_s}\left[1 - \frac{\bar{m}_s}{\bar{m}_t}\right] \end{aligned} \qquad (A.3)$$

The steady state solutions shown above are required by the shell and tube functions $K_S(x)$ and $K_T(x)$ defined in equation (14).

Shellside Equation Galerkin Coefficients

The Galerkin procedure is shown to transform the transient model partial differential equations into a coupled set of ordinary differential equations. The resulting coefficients α_{ij}, β_{ij}, γ_{ij} and F_i in Equation (21) are obtained in terms of known quantities by:

$$\begin{aligned} \alpha_{mn} &= I_3(n,m) \\ \beta_{mn} &= -\alpha_{11}^{o}\frac{n\pi}{2L} I_4(n,m) - \alpha_{12}^{o} I_3(n,m) \\ \gamma_{mn} &= \alpha_{12}^{o}\left[\sin\frac{n\pi}{2} I_4(n,m) - \cos\frac{n\pi}{2} I_3(n,m)\right] \\ F_m &= [I_2(m) - I_1(m)]\left[\frac{d\theta_o}{d\tau} - \alpha_{12}^{o}\theta_o\right] - \frac{\alpha_{12}^{o}\theta_o}{L} I_1(m) - \alpha_{12}^{o}\phi_L I_2(m) + \bar{I}_p(m) \end{aligned} \qquad (A.4)$$

where

$$I_1(m) = \int_0^L \sin\frac{m\pi x}{2L}\,dx = \frac{2L}{m\pi}\left[1 - \cos\frac{m\pi}{2}\right] \qquad (A.5)$$

$$I_2(m) = \int_0^L \frac{x}{L}\sin\frac{m\pi x}{2L}\,dx = \frac{2L}{m\pi}\left[\frac{2}{m\pi}\sin\frac{m\pi}{2L} - \cos\frac{m\pi}{2}\right] \qquad (A.6)$$

$$I_3(n,m) = \int_0^L \sin\frac{n\pi x}{2L}\sin\frac{m\pi x}{2L}\,dx = \begin{cases} \left[\dfrac{\sin (m-n)\pi x/2L}{(m-n)\pi/L} - \dfrac{\sin (m+n)\pi x/2L}{(m+n)\pi/L}\right]_0^L & \text{,when } m \neq n \\ \dfrac{L}{2} & \text{,when } m = n \end{cases} \qquad (A.7)$$

$$I_4(n,m) = \int_0^L \cos\frac{n\pi x}{2L}\sin\frac{n\pi x}{2L}\,dx = \begin{cases} -\left[\dfrac{\cos (m-n)\pi x/2L}{(m-n)\pi/L} - \dfrac{\cos (m+n)\pi x/2L}{(m+n)\pi/L}\right]_0^L & \text{,when } n \neq m \\ \dfrac{L}{2n\pi}[1 - \cos n\pi] & \text{,when } n = m \end{cases} \qquad (A.8)$$

$$I_5(n,m) = \int_0^L \cos\frac{n\pi x}{2L}\cos\frac{m\pi x}{2L}\,dx = \frac{\sin (m-n)\pi/2}{(m-n)\pi/L} + \frac{\sin (m+n)\pi/2}{(m+n)\pi/L} \qquad (A.9)$$

$$\bar{I}_p(m) = \int_0^L K_s(x)\sin\frac{m\pi x}{2L}\,dx \qquad (A.10)$$

Tubeside Equation Galerkin Coefficients

The Galerkin procedure is shown to transform the transient model partial differential equations into a coupled set of ordinary differential equations. The resulting coefficients δ_{ij}, ε_{ij}, μ_{ij}, and G_i in Equation (22) are obtained in terms of known quantities by:

$$\begin{aligned} \delta_{mn} &= J_2(n,m) \\ \varepsilon_{mn} &= -\alpha_{22}^{o} J_4(n,m) \\ \mu_{mn} &= -\alpha_{21}^{o} J_5(n,m) + \alpha_{22}^{o} J_2(n,m) \\ G_m &= -\frac{d\phi}{d\tau} J_1(m) + \alpha_{21}^{o}\frac{f_L}{L} J_3(m) + \alpha_{22}^{o}\theta_o [J_3(m) - J_1(m)] - \alpha_{22}^{o}\phi_L J_1(m) + J_p(m) \end{aligned} \qquad (A\text{-}11)$$

where

$$J_1(m) = \int_0^L \frac{x}{L}\sin\frac{m\pi}{2}\left(1 - \frac{x}{L}\right)dx = \frac{2L}{m\pi}\left[1 - \frac{2}{m\pi}\sin\frac{m\pi}{2}\right] \qquad (A.12)$$

$$J_2(n,m) = \int_0^L \sin\frac{n\pi}{2}\left(1 - \frac{x}{L}\right)\sin\frac{m\pi}{2}\left(1 - \frac{x}{L}\right)dx = \begin{cases} \left[\sin\frac{n\pi}{2}\sin\frac{m\pi}{2} I_5(n,m) - \sin\frac{n\pi}{2}\cos\frac{m\pi}{2} I_4(n,m) - \sin\frac{m\pi}{2}\cos\frac{n\pi}{2} I_4(m,n) + \cos\frac{n\pi}{2}\cos\frac{m\pi}{2} I_3(n,m)\right] & \text{when, } n \neq m \\ \left[\frac{L}{2}\right] & \text{when, } n = m \end{cases} \qquad (A.13)$$

$$J_3(m) = \int_0^L \sin\frac{m\pi}{2}\left(1 - \frac{x}{L}\right)dx = \frac{2L}{m\pi}\left[1 - \cos\frac{m\pi}{2}\right] \qquad (A.14)$$

$$\begin{aligned} J_4(n,m) &= \int_0^L \sin\frac{n\pi x}{2L}\sin\frac{m\pi}{2}\left(1 - \frac{x}{L}\right)dx \\ &= \sin\frac{m\pi}{2} I_4(m,n) - \cos\frac{m\pi}{2} I_3(n,m) \end{aligned} \qquad (A.15)$$

$$J_5(n,m) = -\frac{n\pi}{2L}\int_0^L \sin\frac{m\pi}{2}\left(1 - \frac{x}{L}\right)\cos\frac{n\pi}{2}\left(1 - \frac{x}{L}\right)dx = \begin{cases} \frac{n\pi}{2L}\left[\sin\frac{n\pi}{2}\cos\frac{m\pi}{2} I_3(n,m) - \sin\frac{m\pi}{2}\cos\frac{n\pi}{2} I_5(n,m) + \cos\frac{n\pi}{2}\cos\frac{m\pi}{2} I_4(n,m) - \sin\frac{n\pi}{2}\sin\frac{m\pi}{2} I_4(m,n)\right] & \text{when, } n \neq m \\ -\frac{1}{2}\left(\sin\frac{n\pi}{2}\right)^2 & \text{when, } n = m \end{cases}$$

(A.16)

$$J_p(m) = \int_0^L K_T(x) \sin \frac{n\pi}{2}\left(1 - \frac{x}{L}\right) dx \qquad (A.17)$$

This completes the definition of all Galerkin coefficients required to solve the coupled set of ordinary differential equations for time dependent shell and tube temperature profiles.

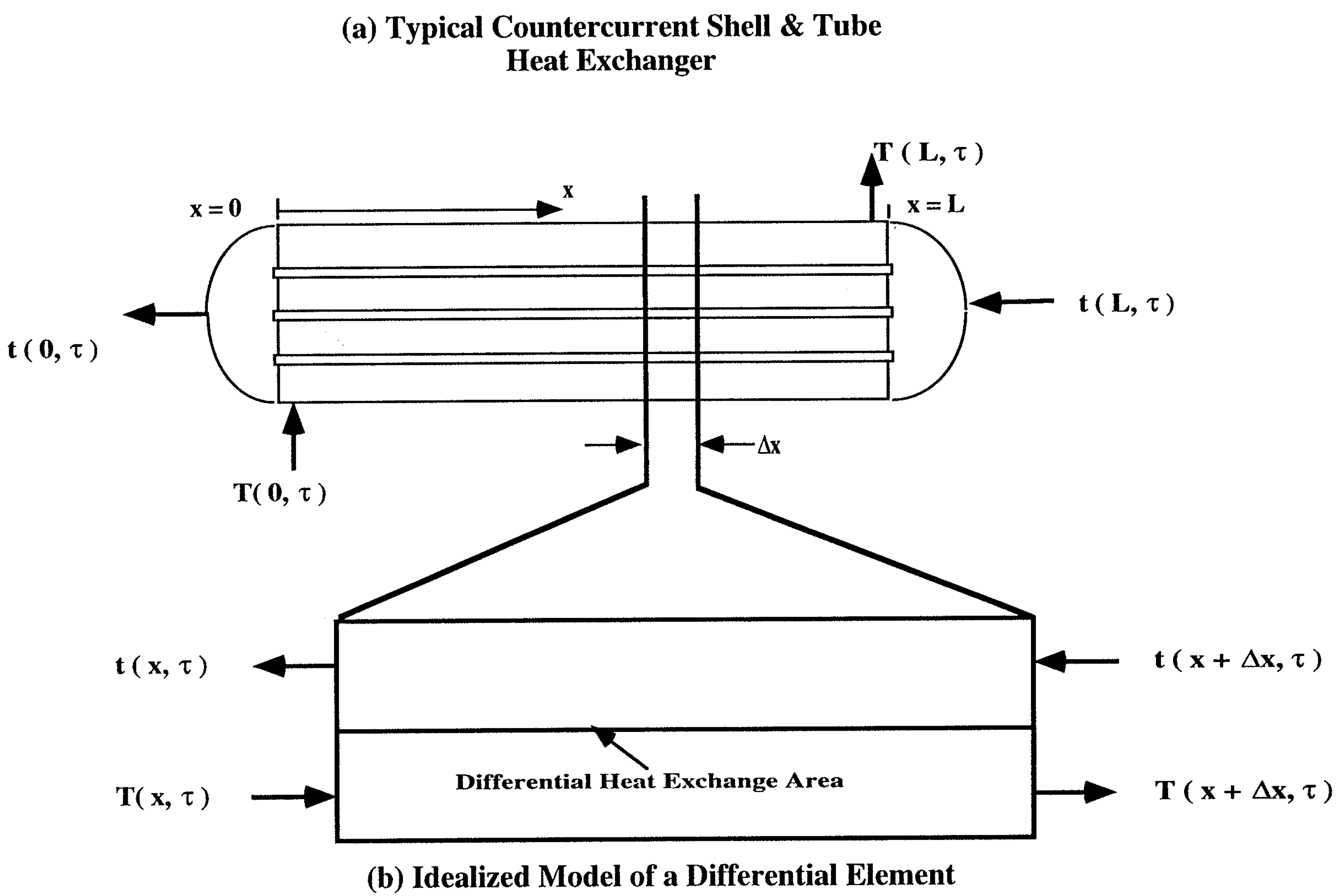

Figure 1: Schematic Outline of the Countercurrent Transient Heat Exchange Process

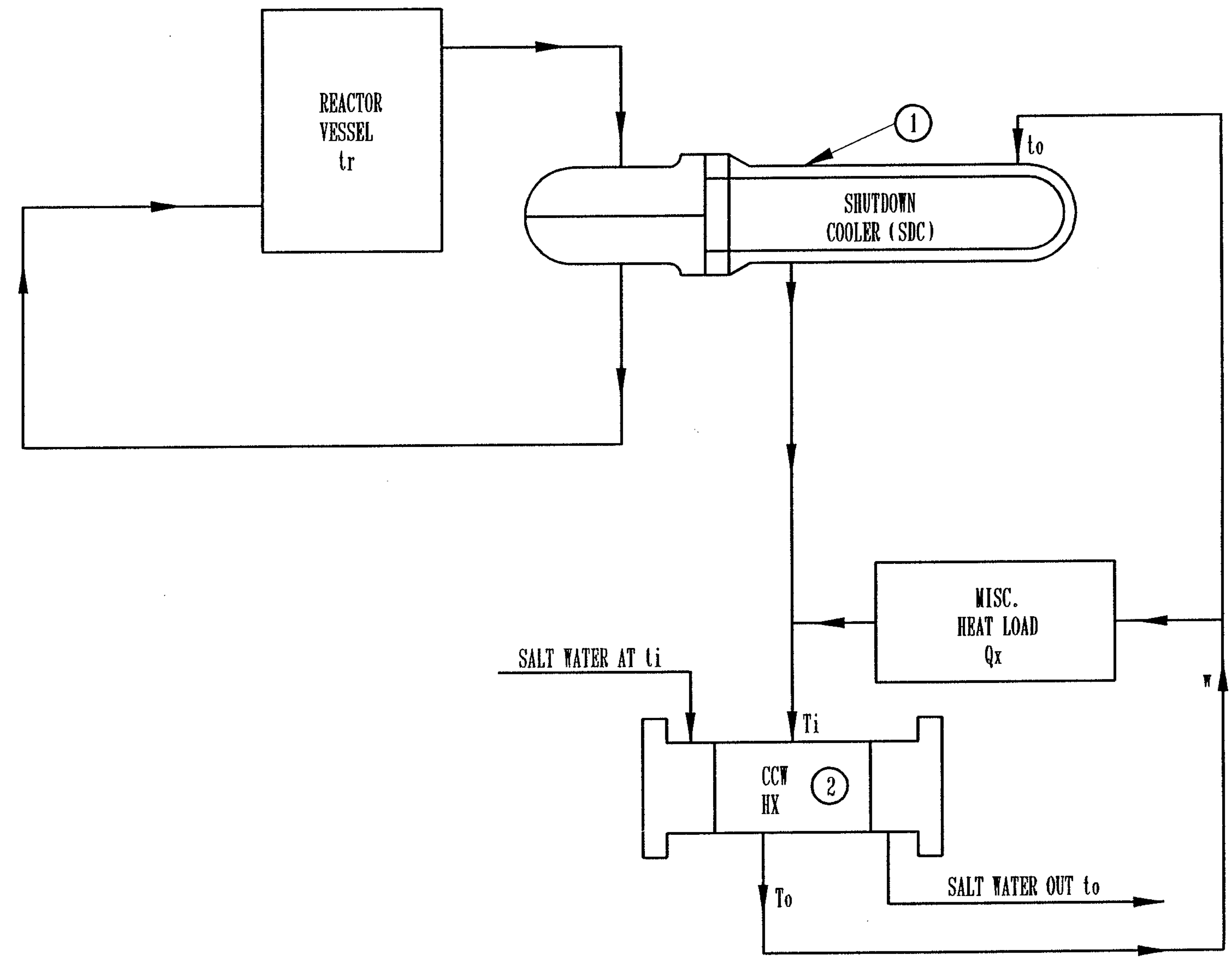

FIGURE 2: SCHEMATIC DIAGRAM OF THE CALVERT CLIFFS COMPONENT COOLING WATER SYSTEM TESTED

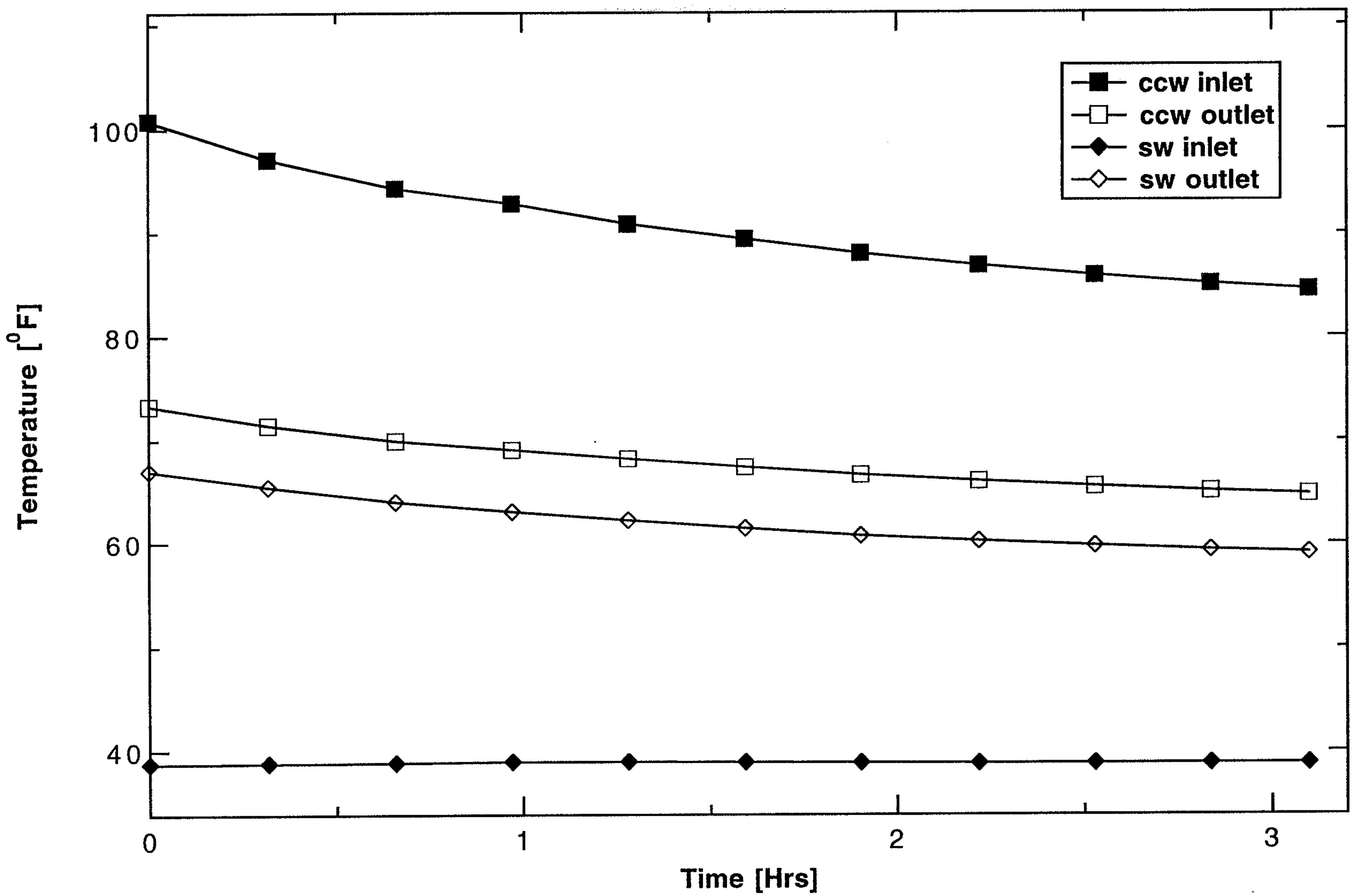

Figure 3: CCWHX Inlet/Outlet Temperature Transients

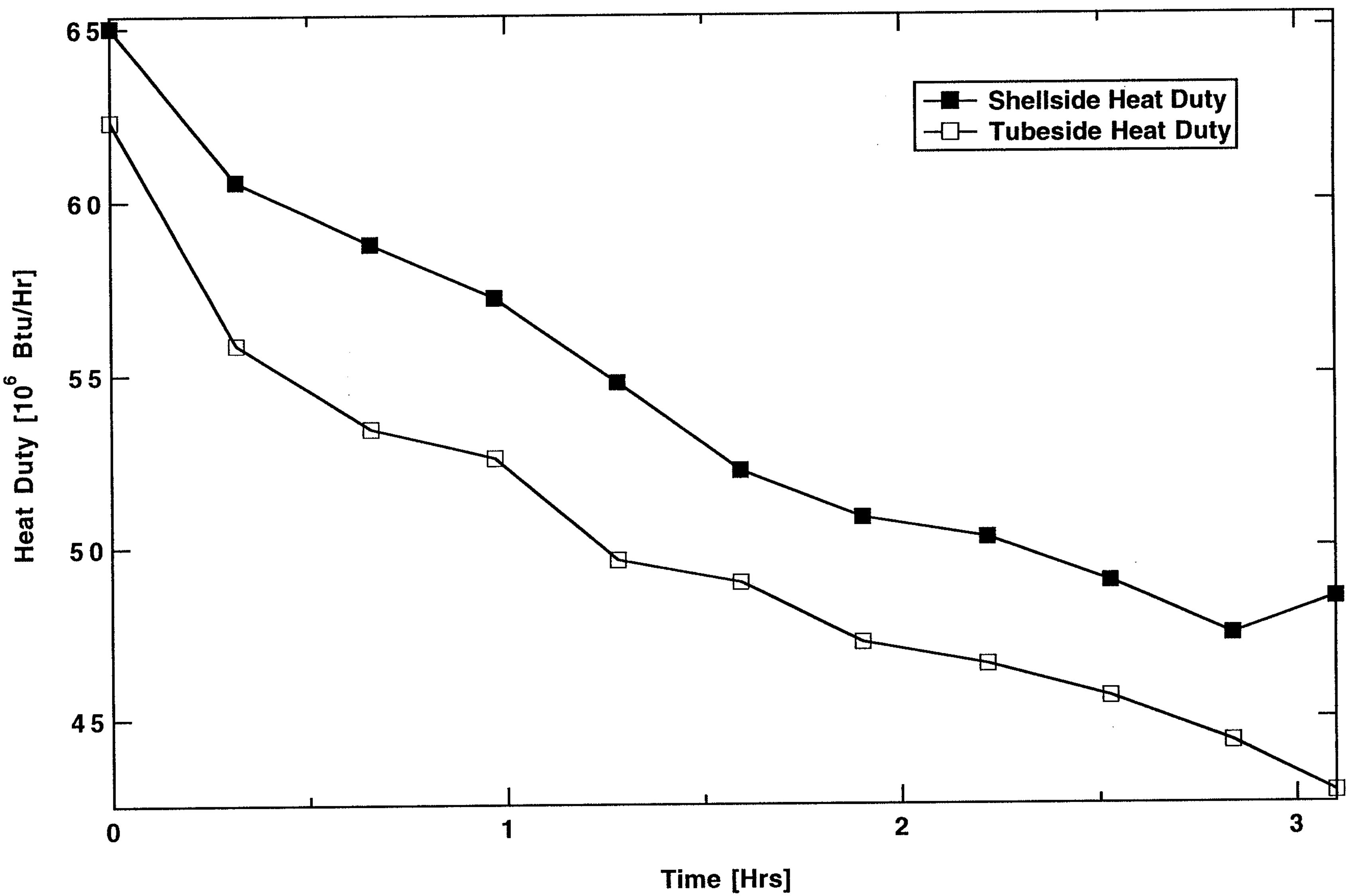

Figure 4: Transient Shell & Tubeside Heat Duty Profiles

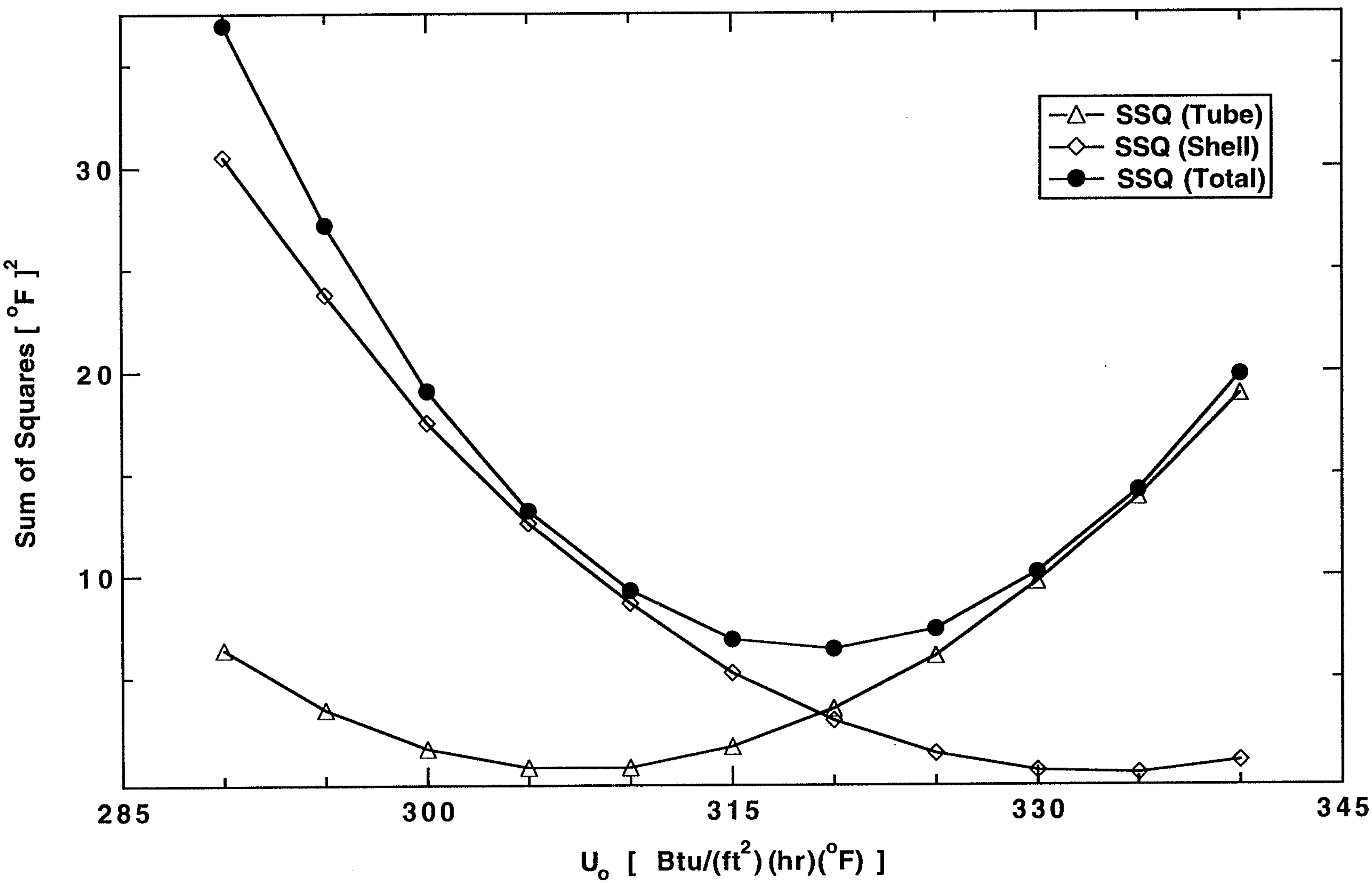

Figure 5: Minimization of Sum of Squares

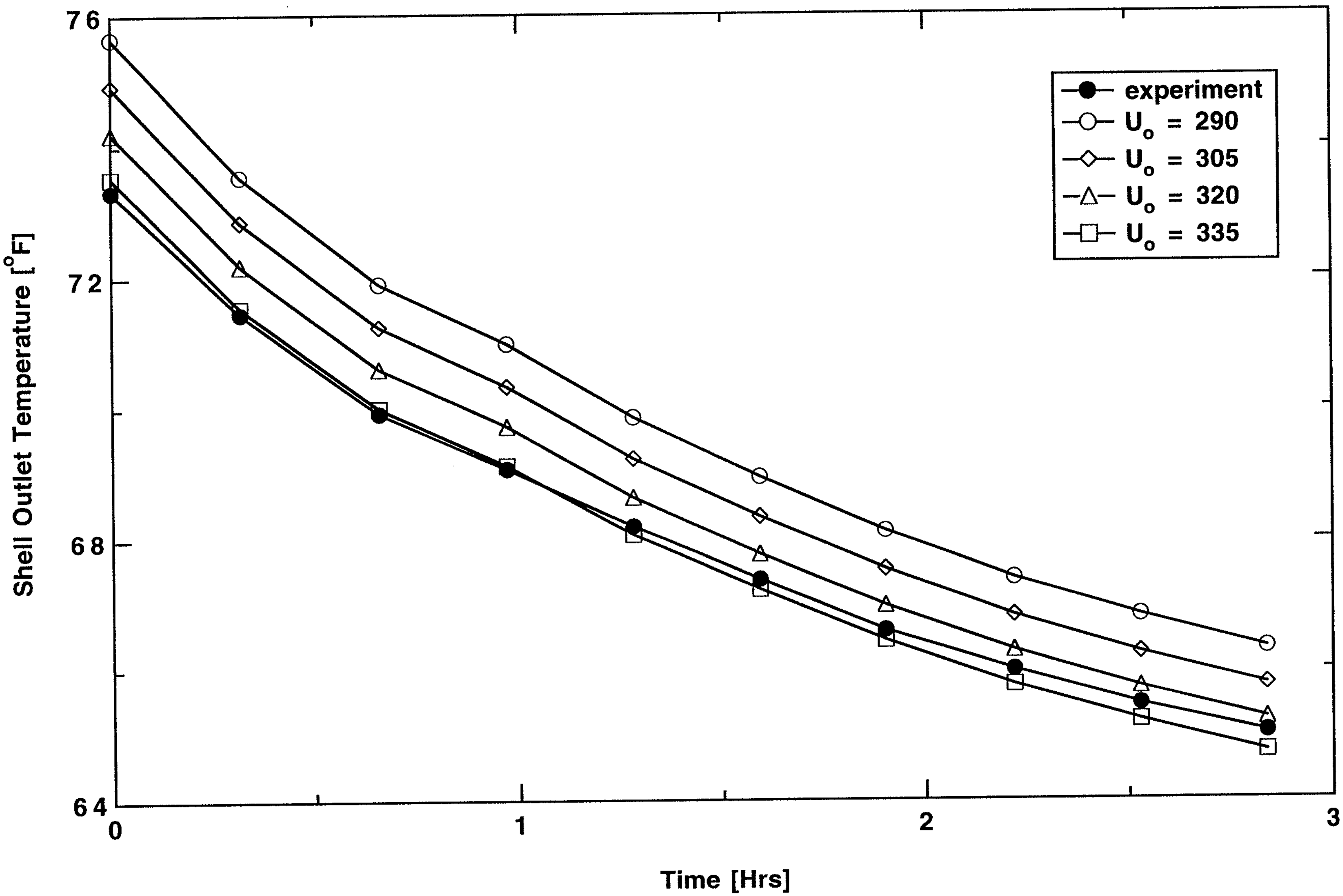

Figure 6: Matching Predicted Temperature Profile to Experiment by Variation of U_o