HEAT TRANSFER CHARACTERISTICS of A GENERALIZED DIVIDED FLOW HEAT EXCHANGER

by

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The concept of a "Divided-flow" heat exchanger is generalized by locating the shell inlet (or outlet) nozzle off-center such that the two shell sub-streams are unequal and traverse unequal flow paths. The governing equations for heat transfer in such an exchanger are derived and solved leading to an optimization problem. In this problem, the optimal subdivision of heat transfer surface to minimize required overall heat transfer surface, under certain restricted conditions, is sought. It is shown that the off-center nozzle location can be selected judiciously so as to maintain (or even improve heat transfer) while reducing the gross shellside pressure loss. Thus, the pumping costs are minimized without sacrificing heat transfer.

1. INTRODUCTION

A divided flow heat exchanger consists of the shell outlet nozzle located at the tube mid-span, and two shell inlet nozzles such that the shell stream is split into two fractions transferring heat with tubeside fluid into two distinct non-overlapping regions. The flow arrangement may also be reversed in some cases such that the inlet is located at the center, and two outlet nozzles are located at the two extremities of the tube bundle span. This exchanger is referred to as J-type in the popular lexicon of the tubular heat exchanger trade [1]. The divided flow exchanger with single tube pass is perhaps the most attractive design from the standpoint of operating costs, since the pressure loss in both shell and tubeside streams are minimized in this configuration. Naturally, it is a highly desirable candidate design in large exchangers handling great amounts of fluid flows. Gardner [2]and Jaw [3] have given expressions for heat transfer characteristics of the standard J-type one tube pass exchanger wherein the shell streams and surface areas are equally divided. However, it can be shown by simple reasoning that locating the outlet nozzle (Fig. 1) off-center reduces the net shell side pressure loss even further. As an added bonus, the overall heat transfer can also be enhanced, in some cases, by a proper selection of the nozzle location. In fact, the nozzle location can be optimized to maximize heat transfer.

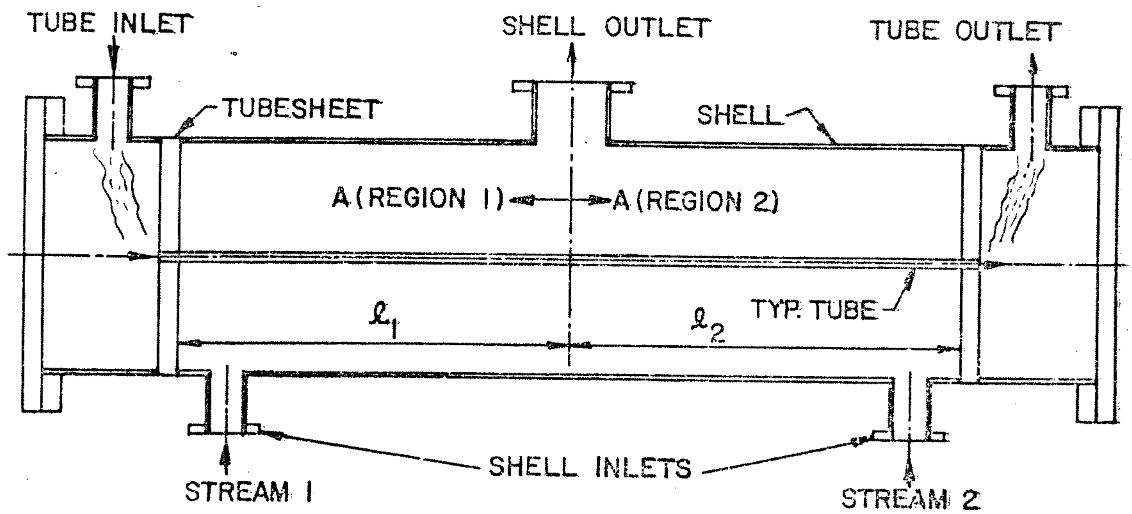

FIGURE 1 - DIVIDED FLOW ONE TUBE PASS HEAT EXCHANGER

In Section 2, an elementary analytical model is used to characterize the attractiveness of off-center nozzle design. An expression relating temperature efficiency P[1, p.144] to the generic parameters of the problem is derived in Section 3. The optimal nozzle location is found using Lagrange multipliers in Section 4. Finally, the import of the analytical solutions derived herein is illustrated through a numerical example in Section 5, followed by concluding remarks in Section 6.

2. PRESSURE LOSS

Referring to Figure 1, the pressure loss Δp_1 in the shell stream in traversing region 1 is proportional to the flow parameter ℓ_1 and square of the flow rate w_1; i.e.

$$\Delta p_1 = k w_1^2 \ell_1 \tag{1}$$

where k depends on the type of baffle geometry and layout. If subregion 2 has identical baffle layout, then

$$\Delta p_2 = k w_2^2 \ell_2 \tag{2}$$

It is recognized that the above expressions are only approximately correct. The pressure loss in a heat exchanger is a highly complex function of a host of parameters; too numerous and varied to enumerate here. However, our object at present is only to establish, in a general sense, the potential superiority of the off-center nozzle design over the standard J-type design. For this purpose a general simplified relationship depicted in Equations (1) or (2) above is quite appropriate.

Since $\Delta p_1 = \Delta p_2$; we have

$$w_1^2 \ell_1 = w_2^2 \ell_2 \tag{3}$$

or

$$w_2 = w_1 \left(\frac{\ell_1}{\ell_2}\right)^{1/2}$$

Hence

$$w = w_1 \left[1 + \left(\frac{\ell_1}{\ell_2}\right)^{1/2}\right] \tag{4}$$

Note

$$\Delta p_1 = k w_1^2 \ell_1$$

or

$$\Delta p_1 = \frac{k w^2 \ell_1}{\left[1 + \left(\frac{\ell_1}{\ell - \ell_1}\right)^{1/2}\right]^2} \tag{5}$$

where ℓ is the total length of tube span.

To find an extremum for Δp_1, we require

$$\frac{\partial \Delta p_1}{\partial \ell_1} = 0$$

This yields

$$1 - \left(\frac{\ell_1}{\ell_2}\right)^{3/2} = 0$$

or

$$\ell_1 = \ell_2$$

Furthermore

$$\frac{\partial^2 \Delta p_1}{\partial \ell^2} = \frac{-3}{2} \left(\frac{\ell_1}{\ell_2{}^3}\right)^{1/2}$$

Hence $\ell_1 = \ell_2$ corresponds to a maximum for Δp_1 (and Δp_2).

Figure 2 shows variation of the dimensionless pressure loss with $(\frac{\ell_1}{\ell})$. We note that $(\frac{\ell_1}{\ell}) \neq 0.5$ always reduces the pressure loss. More unequal ℓ_1 and ℓ_2, lower the pressure loss. However, making ℓ_1 and ℓ_2 unequal has other implications which we will explore later in this paper.

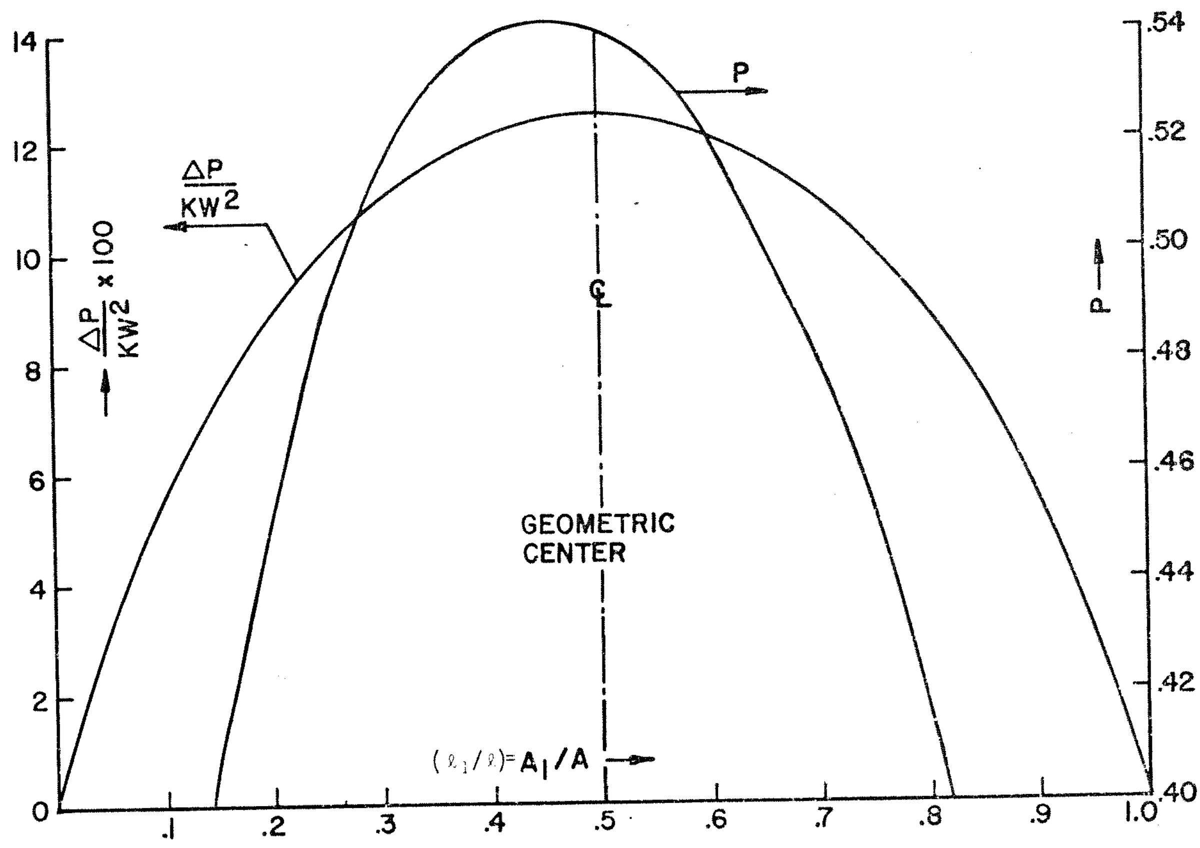

FIGURE 2 - NET PRESSURE LOSS AND TEMPERATURE EFFICIENCY VS NOZZLE LOCATION

3. HEAT TRANSFER RELATIONS

Let us consider a divided flow exchanger shown in Figure 1. Two shell streams of magnitudes w_1 and w_2 enter at the two extremeties of the shell at temperature T_1. They transfer heat with the tubeside fluid (flowing from the left hand channel to the right hand channel); and finally merge at the shell outlet nozzle. We will locate the origin of the surface coordinate in the plane of the shell outlet nozzle. Two independent coordinates, A, define the surface coordinates in the two regions as shown in Figure 1. Let t_1 and t_2 denote tubeside inlet and outlet temperature respectively. Similarly T_2 denotes the shell outlet temperature after the two sub-streams have merged and thoroughly mixed in the outlet nozzle. We will derive the necessary heat transfer relations with the following major assumptions:

a. The heat transfer coefficient is constant in each region. However it can be different in the two regions.
b. The thermal capacities of the streams in each region do not vary along the heat transfer surface.
c. No phase change.
d. Shellside fluid is always thoroughly mixed, and does not undergo appreciable temperature changes in any cross pass.
e. No heat transfer to external environment.

We now proceed to establish the appropriate equations for each region.

I. Region I: Heat transfer across an elemental heat transfer surface area dA yields:

$$-w_t C_{t_1} \, d\alpha = u . dA \, (\theta - \alpha)$$

Where θ and α are shell and tubeside fluid temperatures at a generic coordinate location A.

Thus

$$\frac{d\alpha}{dA} = \frac{-u_1}{w_t C_{t_1}} (\theta - \alpha) \tag{6}$$

$$\frac{d^2\alpha}{dA^2} = \frac{-u_1}{w_t C_{t_1}} \left(\frac{d\theta}{dA} - \frac{d\alpha}{dA}\right) \tag{6.a}$$

Conservation of heat yields

$$-w_1 C_{s_1} d\theta = w_t \, C_{t_1} \, d\alpha$$

or

$$\frac{d\theta}{dA} = \frac{-w_t C_{t1}}{w_1 C_{s1}} \frac{d\alpha}{dA} \tag{7}$$

Substituting for $\frac{d\theta}{dA}$ into equation (6) yields

$$\frac{d^2\alpha}{dA^2} - m\frac{d\alpha}{dA} = 0 \tag{8}$$

where

$$m = \frac{u_1}{w_t C_{t_1}} \left(\frac{w_t C_{t1}}{w_1 C_{s1}} + 1\right) \tag{9}$$

Hence

$$\alpha = B_1 + C_1 e^{mA} \tag{10}$$

where B_1 and C_1 are constants of integration.

II. Region 2: Proceeding as before, we have

$$\frac{d\alpha}{dA} = \frac{u_2}{w_t C_{t2}} (\theta - \alpha) \tag{11}$$

or

$$\frac{d^2\alpha}{dA^2} = \frac{u_2}{w_t C_{t_2}} \left(\frac{d\theta}{dA} - \frac{d\alpha}{dA}\right) \tag{11.a}$$

From heat balance,

$$w_t C_{t_1} \, d\alpha = w_2 C_{s2} \, d\theta \tag{12}$$

Equations (11) and (12) yield

$$\frac{d^2\alpha}{dA^2} - n \frac{d\alpha}{dA} = 0 \tag{13}$$

where

$$n = \frac{u_2}{w_2 C_{t2}} \left(\frac{w_t C_{t2}}{w_2 C_{s2}} - 1\right) \tag{14}$$

Hence

$$\alpha = B_2 + C_2 e^{nA} \tag{15}$$

Where B_2 and C_2 are constants of integration. These, along with B_1 and C_1, are evaluated using the end conditions.

End Conditions:
Note: at $A = A_1$, $(\theta - \alpha) = T_1 - t_1$

Using equations (6) and (10), we have

$$C_1 = \frac{-u_1(T_1 - t_1)}{w_t C_{t1} . m} e^{-mA_1} \tag{16}$$

Furthermore, since at $A = A_1$, $\alpha = t_1$; we have from Equation (10) and (16)

$$B_1 = t_1 + \frac{u_1(T - t_1)}{w_t C_{t1} . m} \tag{17}$$

Thus both constants of integration for region 1 are evaluated.
Continuity of tubeside fluid temperature at A = 0, requires (from equations (10) and (15))

$$B_1 + C_1 = B_2 + C_2$$

or

$$B_2 = t_1 + \frac{u_1(T_1 - t_1)}{w t C_{t1} . m} - \frac{u_1(T_1 - t_1)}{w t C_{t1} . m} e^{-mA_1} - C_2 \tag{18}$$

Moreover, at $A = A_2$; $\theta = T_1$

Note, from Equation (11)

$$\theta = \frac{w_t C_{t2}}{u_2} \frac{d\alpha}{dA} + \alpha$$

Hence

$$T_1 = B_2 + C_2\, e^{nA_2} + \frac{w_t C_{t2} n.}{u_2} C_2 .\, e^{nA_2}$$

Substituting for B_2 from equation (18), and rearranging terms, we have

$$C_2 = \frac{(T_1 - t_1) \{\frac{u_1}{w_t C_{t1} m} - \frac{u_1 e^{-mA_1}}{w_t C_{t1} m} - 1\}}{1 - (1 + \frac{w_t C_{t2} n}{u_2}) e^{nA_2}} \tag{19}$$

Finally, at $A = A_2$; $t = t_2$

Hence

$$t_2 = B_2 + C_2\, c^{nA_2}$$

Substituting for B_2 and C_2 from Equation (19), and rearranging terms we have

$$\frac{(t_2 - t_1)}{(T_1 - t_1)} = \frac{u_1}{w_t C_{t_1} m} - \frac{u_1 e^{-mA_1}}{w_t C_{t_1} m} + \frac{(e^{nA_2} - 1) \{\frac{u_1}{w_t C_{t_1} m} - \frac{u_1}{w_t C_{t_1} m} e^{-mA_1} - 1\}}{[1 - (1 + \frac{w_t C_{t_2} n}{u_2}) e^{nA_2}]} \tag{20}$$

Let $\frac{t_2 - t_1}{T_1 - t_1} = P$

Simplifying Equation (20) further, we have

$$P = \frac{\frac{u_1 n}{u_2 m} (1 - e^{-mA_1})\, e^{nA_2} + e^{nA_2} - 1}{\frac{w_t C_{t2} n}{u_2} e^{nA_2} + (e^{nA_2} - 1)} \tag{21}$$

Equation (21) gives the required relationship between P and the characteristic parameters of the heat transfer problem.

4. OPTIMAL SHELL NOZZLE LOCATION

We seek to minimize the overall required heat transfer surface $(A_1 + A_2)$ by determining the optimal location of the shell outlet nozzle. Stated mathematically, we wish to minimize the function

$$f(A_1, A_2) = A_1 + A_2 \tag{22}$$

Subject to the constraint (Equation 21)

$$g(A_1 A_2) = 0$$

It is assumed here that u_1, u_2, w_1 and w_2 are independent of A_1 and A_2. In other words, the two shell flow rates are independent of the allocation of surface areas between the two subregions. Such an assumption clearly cannot hold rigorously. w_1 and w_2 can, however, be made independent of A_1 and A_2 by suitably arranging the baffle spacings in two subregions. However, the corresponding shellside coefficients will most definitely change. Thus, in order for u_1 and u_2 to remain invariant, the shellside film resistance must be a very small fraction of the overall resistance. It is important to bear these restrictions in mind while using the results derived in this section.
The function g is defined by Equation (20);

$$g = P - \frac{u_1}{w_t C_{t_1} m} + \frac{u_1 e^{-mA_1}}{w_t C_{t_1} m} \tag{23}$$

$$- \frac{(e^{nA_2} - 1)\left(\frac{u_1}{w_t C_{t_1} m} - \frac{u_1}{w_t C_{t_1} m} e^{-mA_1} - 1\right)}{\left[1 - \left(1 + \frac{w_t C_{t_2} n}{u_2}\right) e^{nA_2}\right]}$$

We form the Lagrangian functional

$$\phi = f + \lambda g$$

where λ is the undetermined multiplier. For a minimum, we have

$$\frac{\partial \phi}{\partial A_1} = 0 \tag{24}$$

$$\frac{\partial \phi}{\partial A_2} = 0 \tag{25}$$

Equation (24) and (25) yield

$$\frac{\partial g}{\partial A_1} = \frac{\partial g}{\partial A_2} \tag{26}$$

Performing the necessary differentiations, we have

$$\frac{\partial g}{\partial A_1} = \frac{\frac{u_1 e^{-mA_1}}{w_t C_{t_1}} \; \frac{w_t C_{t_2} n}{u_2} e^{nA_2}}{1 - e^{nA_2} - \frac{w_t C_{t_2} n}{u_2} e^{nA_2}}$$

and

$$\frac{\partial g}{\partial A_2} = \frac{n . e^{nA_2} \; \frac{w_t C_{t_2} n}{u_2} \left(\frac{u_1}{w_t C_{t_1} m} - \frac{u_1}{w_t C_{t_1} m} e^{-mA_1} - 1 \right)}{\left(1 - e^{nA_2} - \frac{w_t C_{t_2} n}{u_2} e^{nA_2}\right)^2}$$

Substituting the foregoing expressions in Equation (26), we have

$$m\left[1 - e^{nA_2}\left(1 + \frac{w_t C_{t_2} n}{u_2}\right)\right] + n\left[1 - e^{mA_1}\left(1 - \frac{w_t C_{t_1} m}{u_1}\right)\right] = 0 \tag{27}$$

Substituting for e^{mA_1} from Equation (27) into Equation (21) yields a quadratic equation in Λ, where $\Lambda = e^{nA_2}$. We have

$$a\Lambda^2 + b\Lambda + C = 0 \tag{28}$$

where

$$a = \frac{a_1 u_1 n}{u_2 m} m - Pa_1{}^2 m + a_1 m$$

$$b = Pa_1 m + Pa_1(m + n) - \frac{u_1 n}{u_2 m}(m + n - na_2) - (m + n) - a_1 m$$

$$c = (m + n)(1 - P)$$

$$a_1 = 1 + \frac{w_t C_{t_2} n}{u_2}$$

$$a_2 = 1 - \frac{w_t C_{t_1} m}{u_1}$$

A_2 follows from Equation (28)

5. EXAMPLE

To illustrate the concepts developed in this paper, we consider the following example problem.

a. Given Data:

(i) Overall surface area, A = 1500 sq. ft.

(ii) Overall heat transfer coefficient (nozzle located on center), u = 200 BTU/Hr.-sq.ft.-^{0}F

(iii) Shellside film coefficient, f_s = 1000 BTU/Hr.Sq.ft.- ^{0}F

(iv) Flow rates:

Tubeside = 200,000 lb/hr.

Shellside = 200,000 lb/hr.

(v) Specific heat (both sides) = 1 BTU/lb-^{0}F

b. Derived data:

We note that the overall coefficient less shellside film coefficient, U_r is given by

$$U_r = \frac{1}{\frac{1}{u} - \frac{1}{f_s}} \tag{28.a}$$

which yields

U_r = 250 BTU/hr. sq. ft.-^{0}F.

Moving the nozzle off-center will result in uneven shell flow streams as defined by Equation (4), wherein the ratio $(\frac{\ell_1}{\ell_2})$ is identical to (A_1/A_2). The corresponding film coefficient is assumed to be proportional to 0.7 exponent of the flow rate; i.e.

$$F_1 = f_s \left(\frac{2w_1}{w_s}\right)^{0.7} \tag{28.b}$$

Thus the overall coefficient in region i is given by

$$u_i = \frac{1}{\frac{1}{u_r} + \frac{1}{F_i}} \; ; \; i = 1,2 \tag{28.c}$$

The foregoing expressions model the variation in u_i fairly accurately for purposes of numerical illustration of the concept proposed herein.

Table 1 shows the value of P, u_1, and u_2 as a function of $\frac{A_1}{A}$. It is to be noted that P, which is a direct measure of heat duty, appears to peak at $\frac{A_1}{A} = 0.45$. P is plotted vs A_1/A in the meaningful range in Figure 2. The value of P corresponding to mid-point nozzle ($A_1/A = 0.5$) is 0.5406. To obtain the same value of P with optimized nozzle location (and reduced A), it is calculated that A should be 1478 sq. ft., a reduction of 22 sq. ft. in the required surface area. Furthermore, the gross pressure loss is reduced by approximately 2.25%. Assuming f_s very large gives the limiting condition wherein the optimal solution developed in the preceding section can be applied. Assuming u = 200, and $w_1 = w_2 =$ 100,000 lb/hr, the optimal values of A_1 and A_2 to produce P = .5406 are found to be 555.1 and 887.5 sq.ft. respectively. Thus the gross heat transfer surface area is 1442.6 sq. ft., compared to 1500 sq. ft. for the symmetrical arrangement.

The gains here, albeit, are rather minor. As a matter of fact, in some cases symmetrical nozzle location is also the most heat transfer effective location. On the other hand, we also found cases where optimization of nozzle location leads to economies in heat transfer surface and pressure loss as high as 10%. In general it can be stated that symmetrical location yields the highest value of heat duty if shellside film resistance is the dominant component in the "overall heat transfer resistance" (inverse of u).

TABLE 1

Variation of P, u_1 and u_2 with nozzle location (Example Problem)

A_1/A	u_1	u_2	P
.950000	166.845	212.255	.218794
.900000	177.793	210.398	.306161
.850000	183.689	208.905	.368302
.800000	187.681	207.572	.415706
.750000	190.694	206.318	.452775
.700000	193.123	205.097	.481904
.650000	195.169	203.880	.504538
.600000	196.951	202.640	.521569
.550000	198.545	201.355	.533513
.500000	200.000	200.000	.540597
.450000	201.355	198.545	.542791
.400000	202.640	196.951	.539313
.350000	203.880	195.169	.531094
.300000	205.097	193.123	.515710
.250000	206.318	190.694	.492253
.200000	207.572	187.681	.458582
.150000	208.905	183.689	.411312
.100000	210.398	177.793	.344530
.050000	212.255	166.845	.245137

6. CONCLUSION

The J-shell exchanger geometry is generalized by varying the location of "Central" shellside nozzle. Thus the two flow regions in which the shell stream divides itself can be made unequal along with unequal tube surface areas. A general expression relating temperature efficiency, P, to the flow, geometry and heat transfer variables is developed. It is shown that the gross shellside pressure drop can be reduced by locating nozzles off-center. Sometimes, the reduction in pressure drop can be even achieved with increased heat transfer!

The optimal nozzle location for a given problem under some restricted conditions, is obtained using Lagrange multipliers.

In general, the optimized asymmetrical arrangement is very advantageous if the LMTD correction factor is low, and shellside film coefficient is not controlling. Similar advantages from non-symmetric nozzle location are shown to be derived in the so called G-type shells (4).

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NOMENCLATURE (PRINCIPAL TERMS):

α Tubeside fluid temperature at a generic coordinate location A

θ Shellside fluid temperature at a generic coordinate location A

λ Lagrange Multiplier

A_1 Heat Transfer area in region 1

A_2 Heat Transfer area in region 2

C_{s1} Specific heat of shellside fluid in region 1

C_{s2} Specific heat of shellside fluid in region 2

C_{t1} Specific heat of tubeside fluid in region 1

C_{t2} Specific heat of tubeside fluid in region 2

$F_1 F_2$ Shellside film coefficients in the two regions

f_s Shellside film coefficient (nozzle mid length)

k Factor of proportionality

ℓ Overall tube length in effective heat transfer

ℓ_1 Tube length in region 1

ℓ_2 Tube length in region 2

P Temperature Efficiency

u Overall Coefficient (shell nozzle located at mid-length)

u_1 Overall Heat Transfer Coefficient in region 1

u_2 Overall Heat Transfer Coefficient in region 2

u_r Overall Coefficient less shellside film Coefficient

w_1 Shell Flow Rate in region 1

w_2 Shell Flow Rate in region 2

w_s Total Shellside Flow Rate

w_t Tubeside Flow Rate