On the Necessary Criteria for Stream-Symmetric Tubular Heat Exchanger Geometries

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The basic criteria are derived in determining whether the heat transfer relationship for a given heat exchanger geometry is invariant to the interchanging of shell-side and tube-side fluids. It is shown that the total heat duty, LMTD correction factor, and other indices of exchanger performance remain unchanged for a stream-symmetric exchanger when the two flow streams are switched, provided the overall heat transfer coefficient is not changed.

INTRODUCTION

Constitutive integrated heat transfer relationships for a tubular heat exchanger are usually expressed in the form

$$\eta = f(P, R) \tag{1}$$

where η, P, and R are the NTU (or reduced thermal flux), temperature efficiency, and thermal flow rate ratio, respectively. These quantities are mathematically expressed as follows:

$$\eta = \frac{UA}{M_t} \tag{2a}$$

$$R = \frac{M_t}{M_s} \tag{2b}$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{2c}$$

f in Eq. (1) implies that η is an implicit function of P and R. The form of f depends on the specific heat exchanger geometry, i.e., the number of shell and tube passes and their relative arrangement. It is of some practical importance to know whether a given heat exchanger geometry is "stream symmetric." In other words, does the functional relationship expressed by Eq. (1) remain unchanged when the tube-side and shell-side fluids are interchanged? As will be shown, the heat duty or the corrected logarithmic mean temperature difference (LMTD) for the stream-symmetric heat exchanger will not change if the shell-side and tube-side stream are switched, provided the overall heat transfer coefficient is assumed to remain unchanged. On the other hand, the performance of a stream-nonsymmetric heat exchanger is dependent on the manner of allocation of the two streams. Clearly, the knowledge regarding the stream symmetry of a design is not a mere idle curiosity. Roetzel and Nicole [1] present a double-series expression for the LMTD which makes implicit use of the concept of stream symmetry. The explicit criteria to ascertain the stream symmetry in a given heat exchanger configuration have not been reported previously in the literature.

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The relationships between the regular and interchanged flow arrangements derived here also help establish their relative merits from the thermodynamic viewpoint. The decision regarding allocation of heat exchanging streams is usually made on the basis of chemical and metallurgical considerations in the process industry. Design pressure and temperature also play a significant role. The high-pressure stream is usually assigned to the tube side for obvious reasons of economy. The importance of the stream allocation on the equipment cost is often overlooked. The stream-symmetric designs are unaffected by the manner of flow assignment. The nonsymmetric design may show wide variation in basic performance parameters (e.g., LMTD correction factor) when the two streams are interchanged. One object of this paper is to point out the significance of stream assignment on the thermal performance of the heat exchangers and hence indirectly on the unit cost.

CRITERIA

Referring to Fig. 1, M_t, t_1, and t_2 represent the tube-side thermal flow rate, tube-side inlet temperature, and tube-side outlet temperature, respectively. M_s, T_1, and T_2 denote the corresponding shell-side quantities. Now if the two streams are interchanged, the corresponding quantities will change to those shown in adjacent parentheses in Fig. 1. Then, by definition, the corresponding expressions of NTU and thermal flow rate ratios are

$$\eta' = \frac{UA}{M_s} \tag{3a}$$

$$R' = \frac{M_s}{M_t} \tag{3b}$$

where prime denotes the corresponding quantities for the interchanged stream configuration. Furthermore, the temperature efficiency P' for the switched-stream arrangement is

$$P' = \frac{T_2 - T_1}{t_1 - T_1} \tag{3c}$$

From Eqs. (2) and (3), we have

$$\eta' = \eta R \tag{4a}$$

$$R' = \frac{1}{R} \tag{4b}$$

$$P' = PR \tag{4c}$$

Moreover, the altered-flow arrangement must also satisfy Eq. (1), i.e.,

$$\eta' = f(P', R') \tag{5}$$

Hence

$$\eta R = f\left(PR, \frac{1}{R}\right) \tag{6}$$

Equation (6) represents a necessary cirterion for stream symmetry. Similarly, if the thermal relationship is expressed with P as an implicit function of η and R,

$$P = \phi(\eta, R)$$

then the corresponding criterion is

$$P' = \phi(\eta', R') = \phi\left(\eta R, \frac{1}{R}\right) = PR \tag{7}$$

It can be readily shown that satisfaction of Eq. (7) ensures that the LMTD correction factor F will also remain invariant to the switching of the two streams.

By definition

$$F = \frac{1}{\eta(R-1)} \ln \frac{1-P}{1-PR} \tag{8}$$

For the switched-flow arrangement,

$$F' = \frac{1}{\eta'(R'-1)} \ln \frac{1-P'}{1-P'R'}$$

Substituting for η', R', and P' from Eq. (4) yields

$$F' = F$$

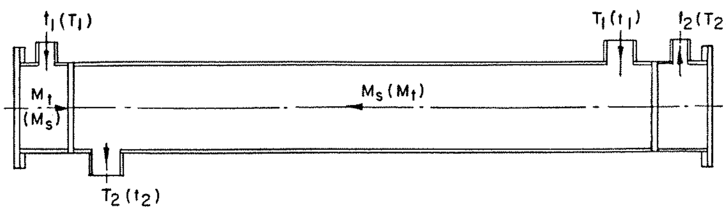

Figure 1 Schematic flow and temperature data for a heat exchanger.

Finally, to demonstrate that the heat duty remains constant when the flow streams are switched in a stream-symmetric heat exchanger, we employ the concept of thermal efficiency as defined by Kays and London [2]. The thermal efficiency ϵ is the ratio of the total heat duty to the thermodynamic maximum heat duty $Q_{\max}$ for identical input quantities. $Q_{\max}$ corresponds to an infinite heat transfer surface. It is shown by Singh [3] that

$$\begin{aligned} \epsilon &= P \qquad R \leqslant 1 \\ &= PR \qquad R \geqslant 1 \end{aligned} \tag{9}$$

We will compute the thermal efficiency ϵ' corresponding to the switched-flow arrangement.

Case (a): $R < 1$

From Eq. (4*b*), $R' = 1/R$; hence, $R' > 1$. Thus

$$\epsilon' = P'R' = PR\,\frac{1}{R} = P = \epsilon$$

Case (b): $R > 1$

Since $R' = 1/R < 1$, then

$$\epsilon' = P' = PR = \epsilon$$

Thus the thermal efficiency and total heat duty are shown to remain unchanged when the tube-side and shell-side fluids are interchanged in a stream-symmetric heat exchanger. The above statement is true as long as the overall heat transfer coefficient is assumed to remain unaltered.

The application of the above criteria to some well-known heat exchanger geometries will now be illustrated.

EXAMPLES

Countercurrent Heat Exchanger

The countercurrent design is obviously stream symmetric. We can formally prove it by noting that the functional relationship for the countercurrent configuration is

$$P = \frac{1 - e^{\eta(R-1)}}{1 - Re^{\eta(R-1)}} \tag{10}$$

If the flow were interchanged, we would have

$$P' = \frac{1 - e^{\eta'(R'-1)}}{1 - R'e^{\eta'(R'-1)}}$$

Substituting for η' and R' in Eq. (9) from Eq. (4), we have

$$P' = \frac{1 - e^{-\eta(R-1)}}{1 - (1/R)e^{-\eta(R-1)}} = PR$$

which satisfies Eq. (7). Hence, the stream symmetry is proved.

One-Shell Pass, Two-Tube Pass

The heat transfer relationship is given by [4, Eq. (28)]

$$P = \frac{2[1 - e^{S\eta}]}{R + 1 - S - e^{\eta S}(1 + R + S)} \tag{11}$$

where

$$S = (1 + R^2)^{1/2} \tag{12}$$

Note that

$$S' = (1 + R'^2)^{1/2} = \frac{S}{R}$$

Thus we have

$$P' = \frac{2(1 - e^{\eta' S'})}{R' + 1 - S' - e^{\eta' S'}(1 + R' + S')}$$

Substituting for η' and R' from Eq. (4), and performing the necessary algebra, we have

$$P' = PR$$

which proves stream symmetry of the one-two design.

Two-Tube-Pass Split-Flow Heat Exchanger

The integrated heat transfer relationships are [4]

$$P = \frac{(2/\eta R)(\theta_1 \ln \theta_1 - \chi \ln \theta_3)}{1 - \theta_1 - 2\chi \ln \theta_3 / R} \tag{13}$$

where

$$\theta_1 = \theta_3 = e^{(-\eta R/2)(1 + 1/2R)} \tag{14a}$$

$$\theta_2 = e^{-\eta R(1-1/2R)} \tag{14b}$$

$$\chi = \frac{1 - \theta_1 - 2 \ln \theta_1/\ln \theta_2 + 2\theta_2 \ln \theta_1/\ln \theta_2 \cdot z}{\theta_3 [1 + (2 \ln \theta_3)/\eta R] - 1} \tag{14c}$$

where $z = 1 + \ln \theta_2/\eta R$

For the altered geometry

$$\theta_1' = e^{-\eta' R'(1-1/2R')} = e^{-\eta R(1-R/2)}$$

$$\theta_2' = e^{-\eta' R'(1-1/2R')} = e^{-\eta R(1-R/2)}$$

It is evident that $\chi' \neq \chi$ and $P' \neq PR$. Hence this geometry is not stream symmetric.

Similarly, it can be shown that split-flow four-tube-pass [5] and divided-flow n-tube-pass heat exchangers ($n = 1, 2, \ldots$) [6, 7] are also stream nonsymmetric.

The effect of the manner of stream allocation on the exchanger thermal performance can be illustrated by considering a typical nonsymmetric design, such as the split-flow two-tube pass. Table 1 gives the values of F and P corresponding to $\eta = 4$ and $R = 0.5$ from [8, p. 153]. The corresponding value of P' for R' [from Eq. (4)] is computed from Eq. (13). We note that F' is over 6% less than F.

This reduced value of F must be compensated for by providing an additional heat transfer surface. The importance of this consideration should not be lost on the system designers.

Finally, it is to be emphasized that the necessary and sufficient condition for stream symmetry is given by Eq. (6) or (7). The so-called LMTD temperature correction factor charts [8, Sec. 9] do not furnish sufficient information to establish stream symmetry. For instance, the value of F for the foregoing example to the switched-stream case ($P' = PR = 0.443$, $R' = 1/R = 2$) is 0.79. This indicates flow symmetry for split-flow two-tube-pass geometry which we know to be incorrect. The erroneous conclusion is reached because the condition $\eta' = \eta R$ is not satisfied by referring to the LMTD correction factor charts alone. Fulfillment of the condition $F'(P', R') = F(P, R)$ is a necessary but not sufficient condition for stream symmetry.

Table 1 Example of a Split-Flow Two-Tube-Pass Geometry

Case	η	R	P	F
Normal	4	0.5	0.885	0.79
Switched stream	2	2.0	0.436	0.74

NOMENCLATURE

A	total effective heat transfer surface
F	LMTD correction factor
M_s	shell-side thermal flow rate (mass flow rate times specific heat)
M_t	tube-side thermal flow rate
P	temperature efficiency
Q	total heat duty
Q_{max}	thermodynamic maximum heat duty
R	thermal flow rate ratio
U	overall heat transfer coefficient
T_1, T_2	shell-side inlet and outlet temperatures, respectively
t_1, t_2	tube-side inlet and outlet temperatures, respectively
η	NTU (or reduced thermal flux)
ϵ	thermal efficiency

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