AN ASME PUBLICATION
$4.00 per copy $2.00 to ASME Members

THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E 47 St., New York, N.Y. 10017

Some Fundamental Relationships for Tubular Heat Exchanger Thermal Performance

Krishna P. Singh
President and CEO
Holtec International
Marlton, NJ

Some basic relationships to characterize tubular heat exchanger thermal performance are derived in terms of the well-known state variables. It is shown that the knowledge of η (NTU), (heat capacity rate ratio), and partial derivatives of the temperature efficiency, P, with respect to η and R enables complete characterization of the exchanger performance around an operating point. Thus, the exchanger performance can be readily predicted for the so-called "subdesign" conditions. Likewise, additional criteria to compare various exchanger styles for a given range of operating conditions can be developed. Two such criteria are developed in this paper.

NOMENCLATURE

A	Total heat transfer surface area, m^2
a	Coefficient of dR^*
b	Coefficient of $d\eta^*$
F	LMTD temperature correction factor, dimensionless
M_s	Shellside heat capacity rate (mass flow rate times the specific heat of shellside fluid), W/K
M_t	Tubeside heat capacity rate, W/K
P	Temperature effectiveness, dimensionless
Q	Total heat duty, W
Q_{max}	Thermodynamic maximum heat duty, W
R	Heat capacity rate ratio, (Eq. 6), dimensionless
T_1, T_2	Shellside inlet and outlet temperatures, respectively, K
t_1, t_2	Tubeside inlet and outlet temperatures, respectively, K
U	Overall heat transfer coefficient, W/m^2K
ε	Exchanger heat transfer effectiveness (Eq. 1), dimensionless
η	Number of heat transfer units (NTU) (Eq. 8), dimensionless
χ	Norm of exchanger effectiveness gradient vector

Contributed by the Heat Transfer Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS for presentation at the 20th Joint ASME/AIChE National Heat Transfer Conference, Milwaukee, Wisconsin, August 2-5, 1981. Manuscript received at ASME Headquarters April 14, 1981.

INTRODUCTION

Heat exchangers are designed to deliver a certain heat transfer rate for a certain specified condition of flow rates and temperatures. This specified condition is often labeled as the "design point" for the heat exchanger [1][1]. However, there are few exchangers which are confined to operating at the design point only. Conditions of service result in variation in the input quantities to the exchanger (namely, flow rates and inlet temperatures) resulting in quite different outlet temperatures and heat duty. Furthermore, the heat exchanger operator may seek to set the input quantities to obtain a desired output. Thus, the "subdesign" characteristics of heat exchangers are undoubtedly of great value to the operator and the system engineer.

A similar motivation for quantifying subdesign characteristics arises when selecting the exchanger style [2] during the design state. If an exchanger is designated to perform under more than one set of conditions, then it is obviously meaningful to compare the candidate designs for their subdesign characteristics rather than focusing the attention exclusively on the design point. In order to make a meaningful comparison between various exchanger geometries "around" a design point, the concept of the "exchanger effectiveness gradient vector" and "LMTD correction factor gradient vector" are proposed. Their practical significance is illustrated by comparing some commonly

1. The brackets denote references listed at the conclusion.

used designs.

In contrast to other process equipment such as pumps and turbines, operating curves for heat exchangers are seldom generated. This is presumably due to the large number of variables involved (two flow rates, four terminal temperatures, heat transfer coefficients and heat transfer surface). However, it is possible to derive explicit relationships to predict exchanger performance when any or some of the principal variables are changed. Such expressions are obviously of value to the user: they also help the designer evaluate a host of operating conditions which ordinarily necessitate use of a computer program.

The expression for the "exchanger effectiveness gradient vector" is derived in the next section, followed by the development of expressions for predicting performance under subdesign conditions. The concept of the "LMTD correction factor gradient vector" is introduced next. These concepts are further illustrated in the succeeding section, followed by a brief conclusion.

EXCHANGER EFFECTIVENESS

There are several definitions of efficiency associated with heat exchanger performance. Of these, "exchanger heat transfer effectiveness" as defined by Kays and London [3] is perhaps most meaningful. The "effectiveness" of a heat exchanger is defined as the ratio of the actual heat duty to the thermodynamically maximum possible heat duty, i.e.

$$\varepsilon = \frac{Q}{Q_{max}} \tag{1}$$

where

$$Q = M_s\ (T_1 - T_2) = M_t\ (t_2 - t_1) \tag{2}$$

and

$$Q_{max} = M_{min}\ (T_1 - t_1) \tag{3}$$

where

$$M_{min} = \text{Min}\ (M_s,\ M_t) \tag{4}$$

ε can be conveniently expressed in terms of commonly used dimensional parameters; namely temperature efficiency P, and heat capacity rate ratio R, defined as follows:

$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{5}$$

$$R = \frac{M_t}{M_s} = \frac{T_1 - T_2}{t_2 - t_1} \tag{6}$$

It can be shown using the foregoing equations that

$$\begin{aligned} \varepsilon &= P;\ R \le 1 \\ &= RP;\ R \ge 1 \end{aligned} \tag{7}$$

Equation (7) leads to some interesting corollaries:

a. Since ε must be less than or equal to 1, it follows that P must <u>always</u> be less than lesser of 1 1/R.

b. The temperature efficiency P is always less than or equal to the "effectiveness" ε.

Another important dimensionless variable that occurs in the formulations of heat exchanger performance analysis is the so-called "Number of Transfer Units NTU" or η,[2] defined as

$$\eta = \frac{UA}{M_t} \tag{8}$$

In fact, it can be shown that η and R can be treated as town independent variables that completely characterize the thermal performance of a heat exchanger; i.e.

$$P = f\ (\eta,\ R) \tag{9a}$$

It thus follows from Eq. (7) that ε can also be considered to be a function of η and R; i.e.

$$\varepsilon = h\ (\eta,\ R) \tag{9b}$$

Thus

$$d\varepsilon = dh = \frac{\partial h}{\partial \eta}\ d\eta + \frac{\partial h}{\partial R}\ dR$$

$$d\varepsilon = dP = \frac{\partial P}{\partial \eta}\ d\eta + \frac{\partial P}{\partial R}\ dR;\ R \le 1 \tag{10}$$

$$\begin{aligned} d\varepsilon &= PdR + RdP \\ &= PdR + R\ (\frac{\partial P}{\partial \eta}\ d\eta + \frac{\partial P}{\partial R}\ dR) \end{aligned}$$

or

$$d\varepsilon = (P + R\ \frac{\partial P}{\partial R})\ dR + R\ \frac{\partial P}{\partial \eta}\ d\eta;\ R \ge 1 \tag{11}$$

Equations (10) and (11) define the incremental change in ε as a function of partial derivatives of P with respect to η and R. We have

$$d\varepsilon = a\ dR^* + b\ d\eta^* \tag{12}$$

where

$$a = \begin{cases} R\ \frac{\partial P}{\partial R};\ R < 1 \\ R\ (P + R\ \frac{\partial P}{\partial R});\ R > 1 \end{cases} \quad \text{(a)}$$

$$b = \begin{cases} \eta\ \frac{\partial P}{\partial \eta};\ R \le 1 \\ \eta R\ \frac{\partial P}{\partial \eta};\ R \ge 1 \end{cases} \quad \text{(b)} \tag{13}$$

and

$$dR^* = \frac{dR}{R} \quad \text{(c)}$$

$$d\eta^* = \frac{d\eta}{\eta} \quad \text{(d)}$$

Thus dR^* and $d\eta^*$ are fractional changes in R and η respectively. Hence, we have

$$d\varepsilon = \nabla\varepsilon .\ \bar{n}$$

where

$$(\bar{n}) = (d\eta^*,\ dR^*) \text{ and}$$

$\nabla\varepsilon$ is the gradient of ε surface in the coordinate system consisting of dR^* and $d\eta^*$. The magnitude of this gradient vector is given by

$$\chi = |\nabla\varepsilon| = (a^2 + b^2)^{\frac{1}{2}} \tag{14}$$

As discussed later, χ can be used as a direct measure of the perturbation in exchanger performance due to small variations in the input variables. We note that a and b are readily computed if the derivatives of P with respect to η and R are known.

Before proceeding to illustrate the usefulness of the above relationships, we will consider similar concepts for another measure of exchanger performance: the so-called "LMTD Temperature Correction Factor".

---

2. Some texts [3] define NTU as UA/M_{min}; where M_{min} is lesser of M_t and M_s. In this paper the definition implied by Eq.(8) is used throughout the text.

TEMPERATURE CORRECTION FACTOR

Whereas ε is a measure of the exchanger effectiveness corresponding to thermodynamically maximum attainable heat transfer rate, the LMTD temperature correction factor F, is the gage of exchanger performance vis-a-vis a countercurrent heat exchanger. It is defined as the ratio of the heat transfer surface area required in a countercurrent heat exchanger to that in the subject heat exchanger for a given set of heat duty and flow conditions. It is expressed in terms of the foregoing parameters η and R as follows:

$$F = \frac{1}{\eta\ (R-1)} \ln \frac{1-P}{1-PR} \tag{15}$$

P appears explicitly in Eq. (15). However, F can also be viewed as an implicit function of η and R by virtue of Eq. (9a).

Derivatives of F with respect to η, P and R are directly obtained from Eq. (15). We have

$$\frac{\partial F}{\partial \eta} = -\frac{1}{\eta^2\ (R-1)} \ln \frac{1-P}{1-PR} = \frac{-F}{\eta} \tag{16}$$

$$\frac{\partial F}{\partial P} = \frac{1}{\eta\ (1-P)\ (1-PR)} \tag{17}$$

$$\frac{\partial F}{\partial R} = \frac{-F}{(R-1)} + \frac{P}{\eta\ (R-1)\ (1-PR)} \tag{18}$$

Furthermore

$$dF = \frac{\partial F}{\partial P} dP + \frac{\partial F}{\partial \eta} d\eta + \frac{\partial F}{\partial R} dR \tag{19}$$

Substituting for dP from Eq. (10), we have

$$dF = \left(\frac{\partial F}{\partial P}\frac{\partial P}{\partial \eta} + \frac{\partial F}{\partial \eta}\right) d\eta + \left(\frac{\partial F}{\partial P}\frac{\partial P}{\partial R} + \frac{\partial F}{\partial R}\right) dR$$

or $$dF = \left(B\frac{\partial P}{\partial \eta} + A\right) d\eta + \left(B\frac{\partial P}{\partial R} + C\right) dR \tag{20}$$

where

$$(A,\ B,\ C) = \left(\frac{\partial F}{\partial \eta},\ \frac{\partial F}{\partial P},\ \frac{\partial F}{\partial R}\right)$$

Thus

$$dF = \eta \left(B\frac{\partial P}{\partial \eta} + A\right) d\eta^* + R\left(B\frac{\partial P}{\partial R} + C\right) dR^* \tag{21}$$

or $$dF = \nabla F.\ \bar{n} \tag{22}$$

∇F is the gradient of F vector in the coordinate space consisting of η^* and R^*. The norm of this vector can be used as a direct measure of the susceptibility of the exchanger performance due to perturbations in the input data. It is to be noted that computation of the norm of ∇F requires evaluating $\partial P/\partial\eta$ and $\partial P/\partial R$ as was the case for $\nabla\varepsilon$ (Eq. 13).

The functional relationship for most heat exchanger configurations have the general form

$$\eta = \phi\ (P,\ R) \tag{23}$$

where ϕ is an explicit function in P and R, $\partial\eta/\partial P$ (and hence $\partial P/\partial\eta$) can be obtained directly from Eq. (23) by differentiation. $\partial P/\partial R$ is computed by observing that

$$\psi\ (\eta,\ P,\ R) = \eta - \phi\ (P,\ R) = 0 \tag{24}$$

Treating ψ to be a function of η and R directly, and also explicitly through P, we have

$$\frac{\partial \psi}{\partial R} + \frac{\partial \psi}{\partial P}\frac{\partial P}{\partial R} = 0 \tag{25}$$

Noting that

$$\frac{\partial \psi}{\partial R} = \frac{-\partial \phi}{\partial R};\ \frac{\partial \psi}{\partial P} = \frac{-\partial \phi}{\partial P}$$

we have

$$\frac{-\partial \phi}{\partial R} - \frac{\partial \phi}{\partial P}\frac{\partial P}{\partial R} = 0$$

or

$$\frac{\partial P}{\partial R} = \frac{-\partial \phi}{\partial R} \Big/ \frac{\partial \phi}{\partial P} \tag{26}$$

$\partial\phi/\partial R$ and $\partial\phi/\partial P$ are directly obtained from Eq. (23). In this manner, the expression for $\partial P/\partial R$ is obtained.

We will now examine some common heat exchanger geometries.

a. Countercurrent Heat Exchanger

The functional relationship is given by [4]

$$\eta = \frac{\ln \frac{1-P}{1-PR}}{R-1} \tag{27}$$

We have by direct differentiation

$$\partial\eta/\partial P = \frac{1}{(1-P)\ (1-PR)} \qquad \text{(a)}$$

$$\partial P/\partial R = \frac{e^{\eta\ (R-1)}\ [\eta\ (R-1) + 1 - e^{\eta\ (R-1)}]}{[1 - R\ e^{\eta\ (R-1)}]^2} \qquad \text{(b)} \tag{28}$$

The norms of $\nabla\varepsilon$ and ∇F can now be directly obtained by substituting above relationships into Eqs. (13) and (20), respectively.

b. One shell pass - two tube pass heat exchangers

The functional relationship is well-known[4].

$$\eta = \frac{1}{(1+R^2)^{\frac{1}{2}}} \ln \left[\frac{2 - P\ [R + 1 - (1+R^2)^{\frac{1}{2}}]}{2 - P\ [R + 1 + (R^2+1)^{\frac{1}{2}}]}\right] \tag{29}$$

This expression can be recast to give P as a function of η and R

$$P = \frac{2\ [1 - e^{\eta R'}]}{R + 1 - R' - e^{\eta R'}\ (1 + R + R')} \tag{30}$$

where

$$R' = (1 + R^2)^{\frac{1}{2}} \tag{31}$$

The derivatives $\partial P/\partial\eta$ and $\partial P/\partial R$ are obtained by direct differentiation. We have, after some manipulation

$$\partial P/\partial\eta = 1 - PR - P + 0.5\ P^2 R \tag{32}$$

$$\frac{\partial P}{\partial R} = -\frac{1}{YR'}\ [2\eta Re^{\eta R'} + P\ \{R'\ (1 - e^{\eta R'}) - R\ (1 + e^{\eta R'}) - \eta Re^{\eta R'}\ (R + 1 + R')\}] \tag{33}$$

where

$$Y = R + 1 - R' - e^{\eta R'}\ (R + 1 + R') \tag{34}$$

c. Two shell pass - four tube pass

The functional relationship is [4]

$$\eta = \frac{2}{R'} \ln \left[\frac{\frac{2}{P} - 1 - R + \frac{2}{P} \left[(1-P)\ (1-PR)\right]^{\frac{1}{2}} + R'}{\frac{2}{P} - 1 - R + \frac{2}{P} \left[(1-P)\ (1-PR)\right]^{\frac{1}{2}} - R'} \right] \quad (35)$$

In this case, evaluation of the partial derivatives is quite tedious. It is preferable to obtain the derivatives numerically on a digital computer, i.e.

$$\frac{\partial \eta}{\partial P} = \lim_{\Delta P \to 0} \frac{\eta(P + \Delta P) - \eta(P)}{\Delta P} \quad (a)$$

$$\frac{\partial \eta}{\partial R} = \lim_{\Delta R \to 0} \frac{\eta(R + \Delta R) - \eta(R)}{\Delta R} \quad (b) \qquad (36)$$

Having determined $\partial \eta/\partial P$ and $\partial \eta/\partial R$; Eq. (26) can now be utilized to evaluate $\partial P/\partial R$.

Prior to illustrating the application of the concepts of effectiveness gradient and LMTD gradient vectors, we will derive general expressions for predicting subdesign performance of tubular exchangers. These expressions are also shown to require evaluation of the same set of partial derivatives, namely $\partial P/\partial \eta$ and $\partial P/\partial R$.

SUBDESIGN RELATIONSHIPS

First of all, we observe that P is independent of temperatures (Eq. 9). It thus follows that P will remain invariant with respect to changes in the inlet or outlet temperatures[3]. Thus, it should be possible to determine exchanger performance for subdesign cases involving changes in temperature data if the value of P for the design point is given. Explicit expressions for the four possible cases are readily obtained as follows:

a. Input temperatures T_1 and t_1 varied (to new values, say $T_1{}^*$ and $t_1{}^*$)[4]:

Noting that

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{t_2{}^* - t_1{}^*}{T_1{}^* - t_1{}^*}$$

We have

$$t_2{}^* = P\ (T_1{}^* - t_1{}^*) + t_1{}^* \quad (37)$$

Heat balance yields

$$R\ (t_2{}^* - t_1{}^*) = T_1{}^* - T_2{}^* \quad (38)$$

This, together with Eq. (37) yield

$$T_2{}^* = T_1{}^* - PR\ (T_1{}^* - t_1{}^*) \quad (39)$$

b. Hot fluid inlet T_1 and cold fluid outlet t_2 varied (to new values $T_1{}^*$ and $t_2{}^*$, respectively)

Equation (37) gives

$$t_1{}^* = \frac{P\ T_1{}^* - t_2{}^*}{P - 1} \quad (40)$$

3. It is presumed that the coefficient U is not affected by temperature changes. For most practical cases, this assumption can be adopted without significant error as long as temperature changes are small.

4. Subdesign temperatures are indicated with a * superscript in this section.

$T_2{}^*$ is expressed in terms of $T_1{}^*$ and $t_2{}^*$ by using Eq. (38) as follows:

$$T_2{}^* = \frac{1}{(P - 1)} \left[(P - 1 + PR)\ T_1{}^* - PR\ t_2{}^*\right] \quad (41)$$

c. Hot fluid outlet T_2 and cold fluid inlet t_1 varied.

Similar analysis is performed to give the following relationship for $T_1{}^*$ and $t_2{}^*$:

$$T_1{}^* = \frac{1}{(1 - PR)} \left[-PRt_1{}^* + T_2{}^*\right] \quad (42)$$

$$t_2{}^* = \frac{1}{(1 - PR)} [P\ T_2{}^* - t_1{}^*\ (PR + P - 1) \quad (43)$$

d. Both outlet temperatures varied:

Given the new values of $T_2{}^*$ and $t_2{}^*$, the associated values of $T_1{}^*$ and $t_1{}^*$ are obtained by a similar procedure. The final results are:

$$T_1{}^* = \frac{t_2{}^*\ RP - (1 - P)\ T_2{}^*}{P + PR - 1} \quad (44)$$

$$t_1{}^* = \frac{P\ T_2{}^* + PR\ t_2{}^* - t_2}{P + PR - 1} \quad (45)$$

Equations (37), (39)----(45) give the eight required relationships to compute two unknown terminal temperatures for each of the four possible cases of combinations of specified temperatures.

e. Flow rates varied:

As opposed to "modest" variation in terminal temperatures, changes in flow rates, and surface area directly affect the values of η and/or R (and also η indirectly through possible changes in U). Thus flow rate changes alter the heat exchanger performance variables in an explicit manner. It is important to know how the exchanger will respond to changes in flow rates within the expected range of variation. Using Taylor series [5], assuming P to be continuous in η and R, we have

$$P\ (\eta^*,\ R^*) = P\ (\eta,\ R) + \left(\frac{\partial P}{\partial \eta} \Delta\eta \quad \frac{\partial P}{\partial R} \Delta R\right)$$

$$+\tfrac{1}{2}\left(\frac{\partial^2 P}{\partial \eta^2} \Delta\eta^2 + 2\ \frac{\partial^2 P}{\partial \eta \partial R} \Delta\eta\ \Delta R + \frac{\partial^2 P}{\partial R^2} \Delta R^2\right) + \text{Terms of higher order} \quad (46)$$

For moderate changes in η and R (ΔR and $\Delta\eta$ small), sufficiently accurate value of P (η^*, R^*) (referred henceforth as P^*) can be obtained by using only the first order derivatives. Thus

$$P^* \doteq P + \frac{\partial P}{\partial \eta} \Delta\eta + \frac{\partial P}{\partial R} \Delta R \quad (47)$$

The partial derivatives $\partial P/\partial \eta$ and $\partial P/\partial R$ are evaluated in accordance with the procedure described in the preceding section. Thus, P^* is computed.

Having determined P^*, the two unknown terminal temperatures can be determined using the appropriate set of equations [out of Eqs. (37)----(45)].

EFFECTIVENESS AND CORRECTION FACTOR GRADIENT VECTORS

It was shown in the foregoing that computation of the partial derivatives of P with respect to η and R directly leads to the determination of $\nabla\varepsilon$ and ∇F.

Evaluation of $\nabla\varepsilon$ and ∇F provide important insight into the stability and sensitivity characteristics of a heat exchanger at the operating point under consideration. Tube pluggings and tube surface fouling are a fact of life in heat exchanger operation. A pertinent question is: how will the heat duty hold up in face of a declining U (fouling) or decreasing surface area (plugged tubes)? The magnitudes of $\nabla\varepsilon$ and ∇F provide direct answers to this question. To verify this fact we refer to Fig. 1 which gives χ (= $|\nabla\varepsilon|$ and ε as a function of η for countercurrent (curve A) and one shell pass-two tube pass (curve B) designs for a specific value of R. We note that χ for "one-two" design is smaller than that for the countercurrent design for the same value of ε. For example, corresponding to $P = \varepsilon = 0.5$, $\chi_A = .312$, $\chi_B = .285$. This indicates that the effectiveness for the "one-two" design will vary less rapidly with respect to changes in input data than the countercurrent design. Table 1 gives numerical values of the derivatives to enable a direct comparison. By way of comparison, let us compute the decline in the exchanger effectiveness for a 1% reduction in U (and hence η) in a countercurrent heat exchanger operating at $P = \varepsilon = 0.5 (R = 0.5)$. Table 1 yields (by interpolation)

$$\partial\varepsilon/\partial\eta = .375$$

Eq. (10) gives

$$d\varepsilon = \eta \frac{\partial\varepsilon}{\partial\eta} d\eta^* = -(.8)(.375)\frac{(.01)}{.8} = -.00375$$

A "one-two exchanger" operating at identical conditions ($P = .5$, $\eta = .867$, $R = .5$), subject to 1% reduction in η has (by interpolation, Table 1)

$$\frac{\partial\varepsilon}{\partial\eta} = .278$$

Hence

$$d\varepsilon = -(.867)(.278)\frac{(.01)}{.867} = -.00278$$

Therefore, the loss of effectiveness in the "1-2" design is 74% of that in a countercurrent design when both designs are subject to 1% reduction in the NTU. Thus the "1-2" design may be considered to be more stable and less sensitive to perturbations in the input data than an identical service countercurrent heat exchanger.

To demonstrate another variation of the same concept, let us investigate the increase in the shellside flow rate required to nullify the loss in heat duty due to a given decrease in NTU. Let us consider the following operating condition:

$$R = 0.5,\ P = 0.565,\ \Delta\eta = -.05$$

The data for the countercurrent design is found from Table 1:

$$\eta = 1.0,\ \frac{\partial\varepsilon}{\partial\eta} = .312,\ \frac{\partial\varepsilon}{\partial R} = -.133$$

From Eq. (10)

$$d\varepsilon = 0 = \frac{\partial\varepsilon}{\partial\eta} d\eta + \frac{\partial\varepsilon}{\partial R} dR$$

Hence

$$dR = \frac{-\frac{\partial\varepsilon}{\partial\eta} d\eta}{\frac{\partial\varepsilon}{\partial R}}$$

Substituting numbers we have

$$dR|_{cc} = -.1173$$

Hence

$$R^* = R + dR = .3827$$

Percentage increase in M_s required

$$= (\frac{R}{R^*} - 1)\ (100) = 30.65\%$$

Similar procedure for "1-2" design yields

$$\eta = 1.1,\ \frac{\partial\varepsilon}{\partial\eta} = .233;\ \frac{\partial\varepsilon}{\partial R} = .187$$

$$dR|_{1-2} = -.0623$$

$$R^* = 0.5 - .0623 = .4377$$

Thus, percentage increase in M_s required is computed to be 14.23%.

This comparison clearly shows the superiority of "1-2" design over its countercurrent counterpart from the standpoint of stability of performance.

Questions like the ones raised above can be readily answered if $\nabla\varepsilon$ is known, which merely implies computing $\partial P/\partial\eta$ and $\partial P/\partial R$.

Evaluation of these derivatives is a simple matter, especially since most design work is performed on digital computers.

The above example mathematically proves what is intuitively obvious. The decline in performance of a countercurrent heat exchanger due to deterioration in U, reduction in A or flow rates is steeper than it is in a "one-two" exchanger. The magnitudes of χ and the components of $\nabla\varepsilon$ mathematically confirm this intuitive result. $\nabla\varepsilon$ is a simple, yet definitive measure of relative sensitivities between two non-countercurrent designs. It is also a convenient tool to predict subdesign performance of operating equipment.

Knowledge of ∇F provides understanding of the equipment response from another vantage point. ∇F for a countercurrent heat exchanger is by definition zero. Large values of ∇F correspond to small value of F. Figures 2 and 3 show ∇F plotted against F for two different heat exchanger geometries. We note that $\nabla F \to 0$ as $F \to 1$ for both designs. This is seen to be true for all heat exchanger configurations. ∇F is thus a second measure of deviation from countercurrency. A sharp decrease in F coincides with a sharp increase in ∇F (Figure 4).

If two exchanger thermal geometries possess identical values of F at the design point, then ∇F provides the basis for judging their relative merits for subdesign conditions. In this sense, ∇F provides a tool for sensitivity studies in the same manner as $\nabla\varepsilon$. Figure 5 shows the coefficients of ∇F vector as a function of P. We note that both derivatives are negative and both increase with P (as F decreases, Ref. Fig.4), although $\partial F/\partial R$ has a well-defined maximum point. These curves point to the plausible conclusion that the LMTD correction factor in inefficient designs respond more sharply to perturbation in input data. Thus, if the heat transfer surface area in a "low F" is reduced, the attendant increase in F will be greater than that in a "high F" unit subject to a similar surface area reduction. On the other hand, the decrease in F due to input data change (such as decreased shellside flow rate) is also liable to be greater in a "low F" geometry compared to a "high F" one. The conditions for each heat exchanger are unique to its function. Therefore, the considerations in regard to subdesign performance will dictate as to how to interpret the coefficients of the LMTD gradient vector, or the effectiveness gradient vector. Our objective herein is merely to formulate their definition and illustrate their possible use. Perhaps not all designs require their evaluation; nonetheless their knowledge certainly yields additional insights into the hardware characteristics.

CONCLUSION

Expressions to evaluate the performance of a heat exchanger under "subdesign" operating conditions have been derived. It has been shown that the partial derivatives $\partial P/\partial\eta$ and $\partial P/\partial R$ are central to the characterization of the exchanger response in the vicinity of an operating point. The "effectiveness gradient vector" or the "temperature correction factor gradient vector" can be readily evaluated if $\partial P/\partial\eta$ and $\partial P/\partial R$ are known. The components of these vectors (in the η - R plane) and their norms yield valuable insight into the exchanger characteristics around the design point. Comparison of these quantities may be useful in selecting the most suitable candidate design for a given set of operating data.

REFERENCES

[1] Standard for Closed Feedwater Heaters, Heat Exchanger Institute, Third Edition (1979)

[2] Standard for Tubular Exchanger Manufacturers Association, Sixth Edition, New York, (1978), p.5.

[3] W.M. Kays and A.L. London, Compact Heat Exchangers, McGraw Hill (1964).

[4] D.Q. Kern, Process Heat Transfer, McGraw Hill (1950)

[5] F.B. Hildebrand, Advanced Calculus for Applications, Prentice Hall (1962), p. 348.

TABLE 1

DERIVATIVES OF P WITH RESPECT TO η AND R†,

COUNTERCURRENT AND "ONE-TWO" HEAT EXCHANGERS (R = 0.5)

NTU, η	COUNTERCURRENT HEAT EXCHANGER			ONE SHELL PASS-2 TUBE PASS HEAT EXCHANGER		
	P	$\frac{\partial\varepsilon}{\partial\eta}$	$\frac{\partial\varepsilon}{\partial R}$	P	$\frac{\partial\varepsilon}{\partial\eta}$	$\frac{\partial\varepsilon}{\partial R}$
0.1	.093	.865	-.425 E-2	.093	.863	-.44 E-2
0.2	.174	.754	-.146 E-1	.173	.748	-.155 E-1
0.4	.307	.587	-.440 E-1	.304	.567	-.492 E-1
0.6	.412	.467	-.763 E-1	.403	.436	-.894 E-1
0.8	.496	.379	-.107	.480	.337	-.130
1.0	.565	.312	-.133	.540	.263	-.169
1.1	.595	.285	-.145	.565	.233	-.187
1.2	.622	.260	-.155	.587	.206	-.205
1.4	.670	.220	-.173	.623	.162	-.236
1.6	.710	.187	-.186	.652	.128	-.264
1.8	.745	.160	-.196	.675	.101	-.288

†Note: $\frac{\partial P}{\partial\eta} = \frac{\partial\varepsilon}{\partial\eta}$; $\frac{\partial P}{\partial R} = \frac{\partial\varepsilon}{\partial R}$ (from Eq.10) for R $\leq$ 1.0

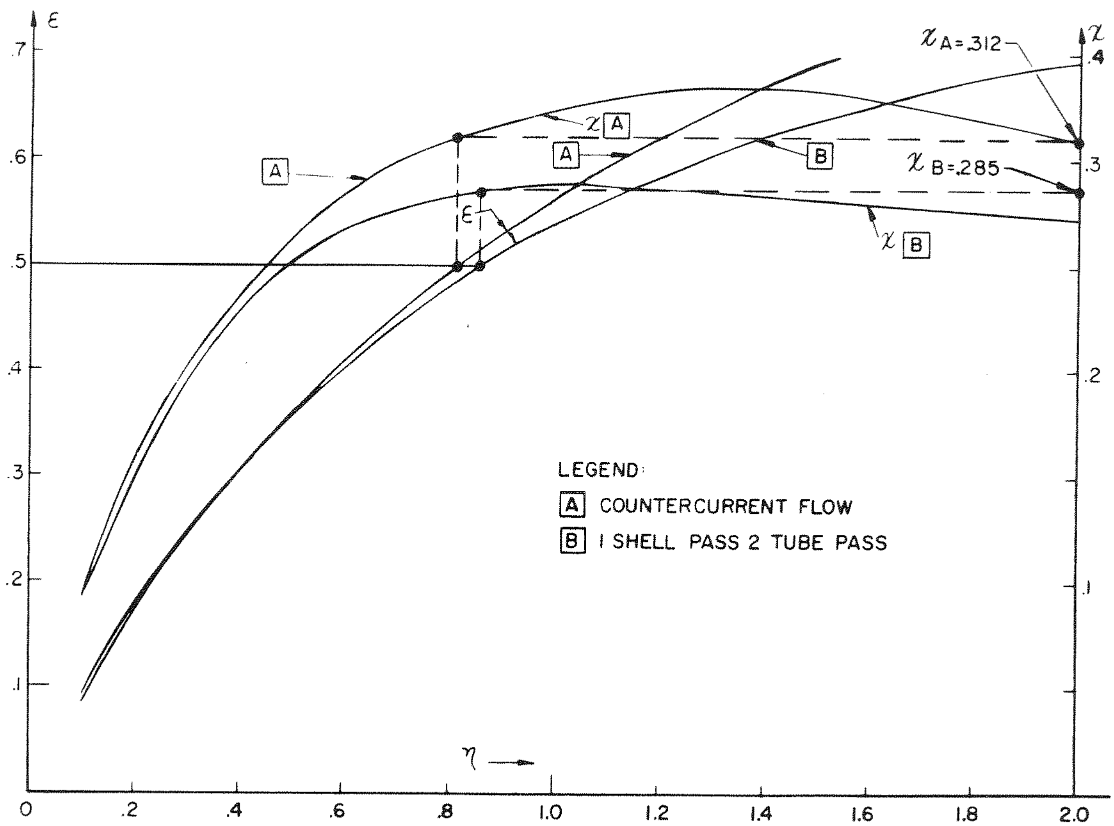

FIGURE 1: ε and χ vs η ($\dot{R}$ = 0.5)

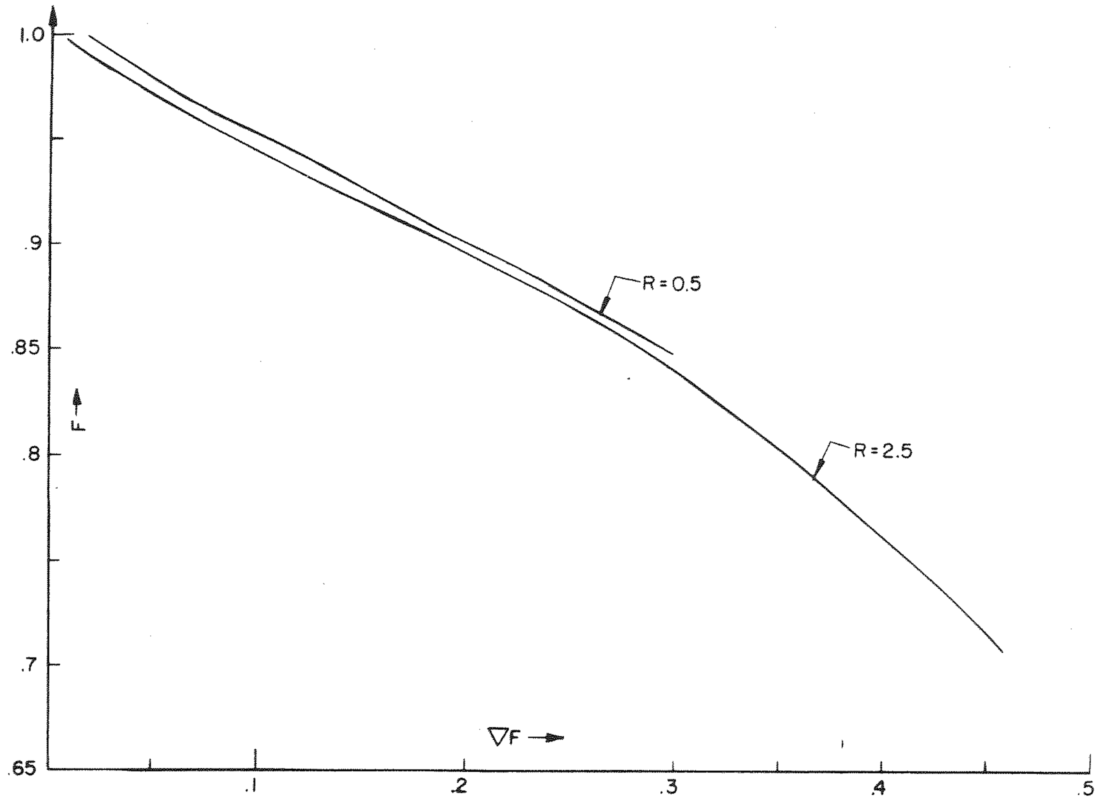

FIGURE 2: F vs ∇F for 1 Shell Pass-2 Tube Pass Heat Exchanger

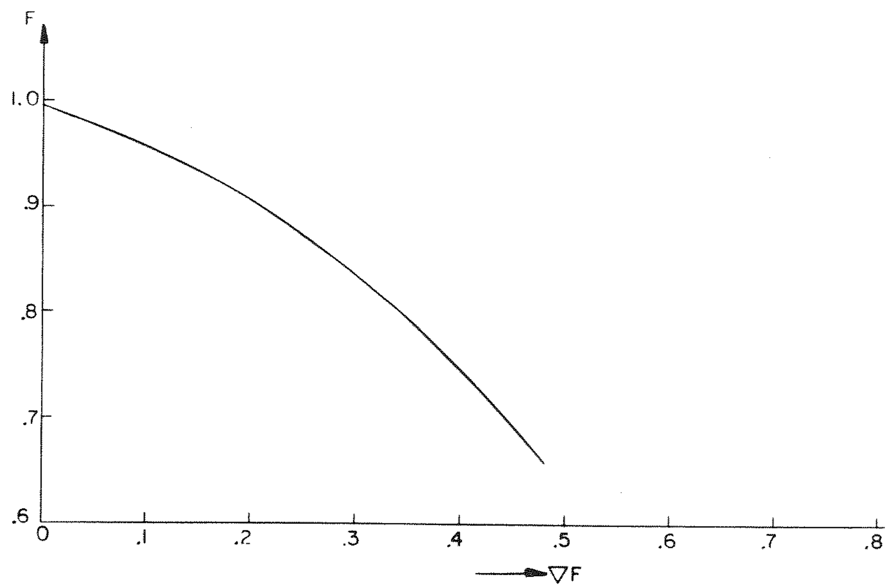

FIGURE 3: ∇F vs F; 2 Shell Pass-4 Tube Pass (R = 0.5)

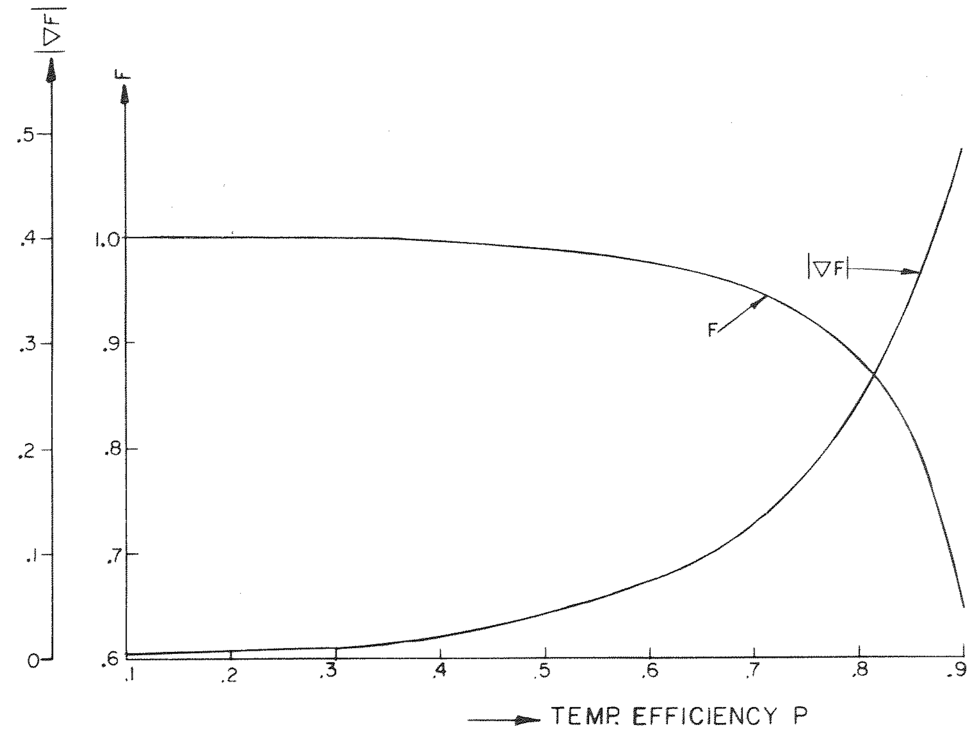

FIGURE 4: F and |∇F| vs P (2 Shell Pass-4 Tube Pass HX)

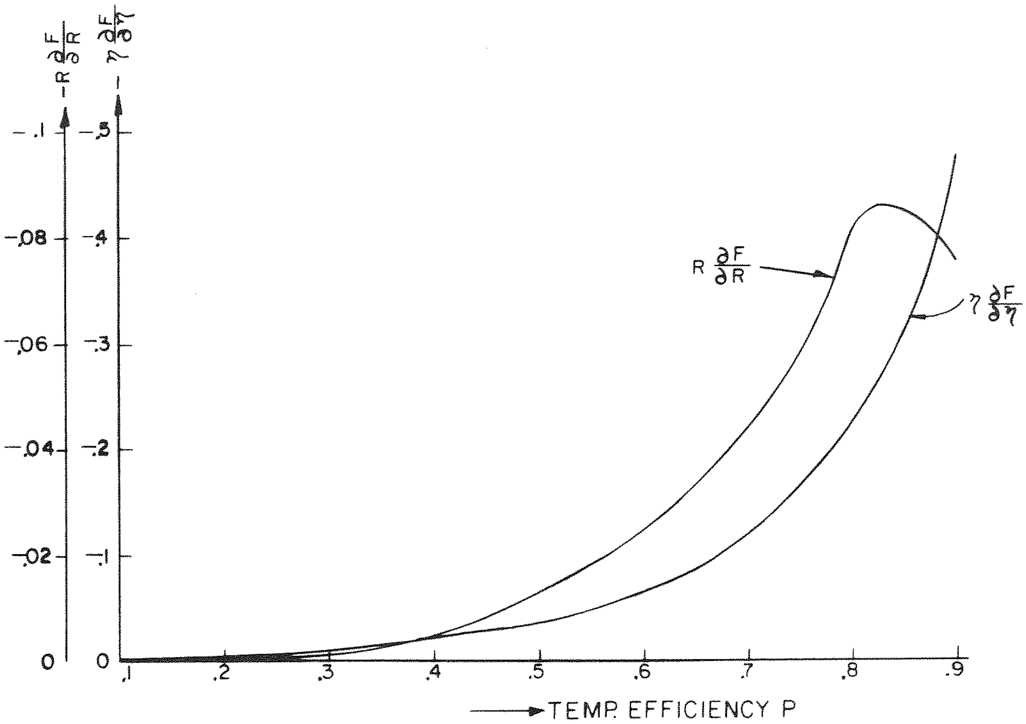

FIGURE 5: Derivatives of F vs P (2 Shell Pass-4 Tube Pass HX)